IRS-aided NOMA for Rate Fairness in Downlink Networks

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Abstract—This paper studies an IRS-aided non-orthogonal multiple access (NOMA) downlink (DL) system, where the design problem aims at improving the rate fairness among all users. This leads to a non-convex problem due to coupling between beamforming vectors and IRS phase shift. We thus propose an IA-ADMM-based algorithm to solve the problem. The effectiveness of the proposed algorithm is verified by numerical results.

I. INTRODUCTION

Intelligent reflecting surface (IRS), in which a reconfigurable elements enable to improve the channel gain by reflecting the incoming signal with the low power consumption, is a promising technique to improve both spectral and energy efficiencies [1]. Meanwhile, non-orthogonal multiple access (NOMA) has recently attracted a lot of interest since it allows users (UEs) to share the same resources [2]. Accordingly, the authors in [3] have considered a NOMA IRS-aided downlink (DL) system, in which beamforming and IRS phase shift are optimized alternatively to minimize the power consumption. However, the design for IRS and NOMA-based beamforming in terms of max-min rate (MMR) has not been exploited in the literature.

In this paper, we investigate an IRS-aided NOMA DL system, where the NOMA-based beamforming and IRS phase shift are jointly optimized to enhance MMR among all UEs. The resulting problem is non-convex, and thus, we utilize a combination of the alternating direction method of multipliers (ADMM) and inner approximation (IA) method to decompose the original problem into two sub-problems. Consequently, NOMA-based beamforming vectors and IRS phase shift are alternatively updated to obtain the local optimal solution at the convergence.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As illustrated in Fig. 1, we consider an IRS-aided DL system, consisting of $K = 3$ single-antenna UEs, with $\mathcal{K} = \{1,...,K\}$, served by one $L$-antennas BS and one $N$-elements IRS. The signal received at UE$_k$ can be expressed as

$$ y_k = \sum_{l \in \mathcal{K}} (h_{k,l}^H + g_{k,l}^H \Phi F) w_l x_l + n_k, \quad k \in \mathcal{K},$$

(1)

where $x_k$ is the symbol intended to UE$_k$, with $\mathbb{E}\{|x_k|^2\} = 1$, $h_{k,l} \in \mathbb{C}^{L \times 1}$, $g_{k,l} \in \mathbb{C}^{N \times 1}$ and $F \in \mathbb{C}^{N \times L}$ denotes the channel links of BS $\rightarrow$ UE$_k$, IRS $\rightarrow$ UE$_k$ and BS $\rightarrow$ IRS, respectively.

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III. PROPOSED ITERATIVE ALGORITHM

We propose an ADMM-based algorithm, providing a local optimal solution for problem (5). With the feasible point $(w^{(i)}, \Phi^{(i)})$ at $i$-th iteration, the ADMM method can be applied as alternatively updating $w$ and $\Phi$ as follows.
A. Update of w

To tackle constraint (5b), we utilize [4, Eq. (B.1)] to respectively obtain the lower bounds of (2a), (3a) and (3b) with respect to \( w \) as

\[
\begin{align*}
A_1(w) - A_2(w) & \geq \lambda, \\
B_{1,k}(w) - B_{2,k}(w) & \geq \lambda, \\
C_{1,t,k}(w) - C_{2,t,k}(w) & \geq \lambda,
\end{align*}
\]

where \( A_1(w), A_2(w), B_{1,k}(w), B_{2,k}(w), C_{1,t,k}(w) \) and \( C_{2,t,k}(w) \) are defined as

\[
\begin{align*}
A_1(w) & \triangleq 2 \frac{\chi_{1,1}(w, \Phi^{(i)})}{\psi_{1,3}(w^{(i)}, \Phi^{(i)}) + \sigma_i^2}, \\
A_2(w) & \triangleq \frac{\chi_{1,1}(w^{(i)}, \Phi^{(i)})}{(\psi_{1,3}(w^{(i)}, \Phi^{(i)}) + \sigma_i^2)^2} (\psi_{1,3}(w, \Phi^{(i)}) + \sigma_i^2), \\
B_{1,k}(w) & \triangleq 2 \frac{\chi_{k,k}(w, \Phi^{(i)})}{\Theta_k}, \\
B_{2,k}(w) & \triangleq \frac{\chi_{k,k}(w^{(i)}, \Phi^{(i)})}{(\sum_{\forall \not= k} \psi_{k,k}(w, \Phi^{(i)}) + \sigma_k^2)}, \\
C_{1,t,k}(w) & \triangleq 2 \frac{\chi_{t,k}(w, \Phi^{(i)})}{\Xi_{t,k}}, \\
C_{2,t,k}(w) & \triangleq \frac{\chi_{t,k}(w^{(i)}, \Phi^{(i)})}{(\sum_{\forall \not= k} \psi_{t,k}(w, \Phi^{(i)}) + \sigma_k^2)},
\end{align*}
\]

in which \( \Theta_k \triangleq \sum_{\forall \not= k} \psi_{k,k}(w, \Phi^{(i)}) + \sigma_k^2 \) and \( \Xi_{t,k} \triangleq \sum_{\forall \not= k} \psi_{t,k}(w, \Phi^{(i)}) + \sigma_k^2 \).

B. Update of \( \Phi \)

Similarly to (6), the lower bounds of (2a), (3a) and (3b) for updating \( \Phi \) can be formulated respectively as

\[
\begin{align*}
\tilde{A}_1(\Phi) - \tilde{A}_2(\Phi) & \geq \lambda, \\
\tilde{B}_{1,k}(\Phi) - \tilde{B}_{2,t,k}(\Phi) & \geq \lambda, \\
\tilde{C}_{1,t,k}(\Phi) - \tilde{C}_{2,t,k}(\Phi) & \geq \lambda,
\end{align*}
\]

where \( \tilde{A}_1(\Phi), \tilde{A}_2(\Phi), \tilde{B}_{1,k}(\Phi), \tilde{B}_{2,t,k}(\Phi), \tilde{C}_{1,t,k}(\Phi) \) and \( \tilde{C}_{2,t,k}(\Phi) \) are defined respectively as

\[
\begin{align*}
\tilde{A}_1(\Phi) & \triangleq 2 \frac{\chi_{1,1}(w^{(i)}, \Phi^{(i)})}{\psi_{1,3}(w^{(i)}, \Phi^{(i)}) + \sigma_i^2}, \\
\tilde{A}_2(\Phi) & \triangleq \frac{\chi_{1,1}(w^{(i)}, \Phi^{(i)})}{(\psi_{1,3}(w^{(i)}, \Phi^{(i)}) + \sigma_i^2)^2} (\psi_{1,3}(w^{(i)}, \Phi^{(i)}) + \sigma_i^2), \\
\tilde{B}_{1,k}(\Phi) & \triangleq 2 \frac{\chi_{k,k}(w^{(i)}, \Phi^{(i)})}{\Theta_k}, \\
\tilde{B}_{2,k}(\Phi) & \triangleq \frac{\chi_{k,k}(w^{(i)}, \Phi^{(i)})}{(\sum_{\forall \not= k} \psi_{k,k}(w^{(i)}, \Phi^{(i)}) + \sigma_k^2)}, \\
\tilde{C}_{1,t,k}(\Phi) & \triangleq 2 \frac{\chi_{t,k}(w^{(i)}, \Phi^{(i)})}{\Xi_{t,k}}, \\
\tilde{C}_{2,t,k}(\Phi) & \triangleq \frac{\chi_{t,k}(w^{(i)}, \Phi^{(i)})}{(\sum_{\forall \not= k} \psi_{t,k}(w^{(i)}, \Phi^{(i)}) + \sigma_k^2)}.
\end{align*}
\]

IV. Numerical Results

We consider a NOMA DL system aided by an IRS with \( K = 3 \) UEs, one BS equipped with \( L = 2 \) antennas and one IRS with \( N = 50 \) elements, as illustrated in Fig. 1. The path loss (in dB) of the links from BS to UEs is modeled as \( P_{\text{BS}} = 30 + 37.5 \log_{10}(d) \), while that of the links connected to IRS is \( P_{\text{IRS}} = 30 + 22 \log_{10}(d) \). The noise power spectral density is set to \(-174\) dBm/Hz. To evaluate the performance of the proposed algorithm, we examine two existing methods: Beamforming and NOMA without (w/o) IRS.

Fig. 2 shows the MMR as a function of \( P_{\text{BS}}^{\text{max}} \). Overall, Alg. 1 provides the highest MMR when compared to the other two schemes. Remarkably, the NOMA-based methods achieve an increasing performance gain along with \( P_{\text{BS}}^{\text{max}} \), as referenced to the beamforming design. In particular, at \( P_{\text{BS}}^{\text{max}} = 18 \) dBm, the performance gains of Alg. 1 and NOMA w/o IRS are respectively 1 and 0.5 bps/Hz, while those increase to 2.5 and 2 bps/Hz at \( P_{\text{BS}}^{\text{max}} = 22 \) dBm. This demonstrates the effectiveness of the proposed algorithm with a joint optimization of NOMA-based beamforming and IRS technique.

V. Conclusion

We have proposed an IRS-aided NOMA-beamforming design, where an MMR optimization problem is considered. To solve the non-convex problem, we have proposed iterative algorithm that combines ADMM and IA methods. Numerical results have been provided to verify that the proposed algorithm outperforms existing schemes.

REFERENCES