3D Moving Target Tracking Using TDoA and FDoA Measurements based on SIR Particle Filter

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Abstract
The accuracy of outdoor moving-target tracking systems usually suffers from ranging errors, sensor position, and velocity errors caused by low SNR (signal noise ratio) and non-line-of-sight (NLOS) conditions. In this paper, we propose a new method for moving-target tracking using joint time-difference-of-arrival (TDoA) and frequency difference-of-arrival (FDoA) measurements. In this work, we introduce particle filter to fuse the interval measurement of TDoA and FDoA for estimating the position and velocity of a moving target. The particle filter plays a key role in the field of moving target detection and estimation, such as unmanned aerial vehicles (UAVs). The estimated interval must contain the actual values of the position and velocity of the moving target. The simulation is expected to improve the accuracy of the positioning performance while minimizing the sensor position and velocity error.

I. INTRODUCTION
With the recent development of unmanned vehicles (UAVs), research on the demand for non-cooperative location estimation and precision improvement is being actively conducted. Methods such as time of arrival, time difference of arrival (TDoA), angle of arrival, two way ranging, frequency difference of arrival (FDoA), etc., exist as a method of tracking the location of UAVs [1]. Recently, for accurate location estimation, a hybrid positioning method capable of obtaining more information with a few receivers using two or more measured values has been studied.

The TDoA/FDoA fusion positioning method is used to estimate the positioning and velocity of moving objects, and in the process of fusing each measured value of TDoA and FDoA, there is a problem of poor positioning accuracy due to problems such as positioning errors and system noise [2]. The particle filter proposed in this paper is a non-linear filter applicable to systems that do not follow a normal distribution and is suitable for estimating the actual position of an object moving by estimating the position using the measured in a noisy environment [3]. This paper proposes a particle filter based on TDoA/FDoA measurement fusion positioning method and presents the simulation results.

II. SYSTEM MODEL
A. Mobility Model
First, the simulation is assumed to be a 3D Euclidean space. A total of four receiver sensors are placed at designated coordinates and track the position and speed of the moving object. The state of the target \(x_k \in \mathbb{R}^{9 \times 1}\) at time \(t_k\) is given as [4]

\[
x_k = \Phi x_{k-1} + \Gamma w_{k-1},
\]

where \(x_k = [u_k^T, \dot{u}_k^T, \ddot{u}_k^T]^T\) consist of the 3D position vector \(u_k\), the velocity vector \(\dot{u}_k\), and the acceleration vector \(\ddot{u}_k\)

\[
u_k = [x_k^T, \dot{x}_k^T, \ddot{x}_k^T, \dot{y}_k^T, \ddot{y}_k^T, \dot{z}_k^T, \ddot{z}_k^T].
\]

The state transition matrix \(\Phi\) is

\[
\Phi = \begin{bmatrix}
I_{3 \times 3} & \Delta I_{3 \times 3} & \Delta^2 I_{3 \times 3} / 2 \\
0_{3 \times 3} & I_{3 \times 3} & \Delta I_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3} & \alpha I_{3 \times 3}
\end{bmatrix},
\]

where \(I_{n \times n}\) is an \(n \times n\) identity matrix, \(0_{n \times n}\) is a \(n \times n\) zero matrix and \(\alpha\) is a constant acceleration parameter. The transformation matrix of the process noise, \(\Gamma\), is

\[
\Gamma = \begin{bmatrix}
\Delta^2 I_{3 \times 3} / 2, & \Delta I_{3 \times 3}, & I_{3 \times 3}
\end{bmatrix}^T,
\]

where \(\Delta = t_k - t_{k-1}\) is a fixed time step and \(w_{k-1}\) is the following white Gaussian noise process:

\[
w_{k-1} = [w_{x,k-1}, w_{y,k-1}, w_{z,k-1}]^T,
\]

\[
E[w_{k-1}] = 0, \quad E[w_{k-1}w_{k-1}^T] = Q_w,
\]

where \(w_{k-1}\) is a zero-mean white Gaussian process noise and the covariance matrix \(Q_w = \sigma_w^2 I_{3 \times 3}\) and \(\sigma_w\) is the standard deviation of the process noise. Let \(r_{i}\) be the true distance between the source and receiver \(i\), i.e.,

\[
r_i(x_k) = ||u_k - s_i||,
\]

where \(u_k\) is the position vector of the target and \(||\cdot||\) represents the 2-norm. The TDoA measurements between receiver \(i\) and receiver \(1\) is

\[
r_{i1}(x_k) = c(t_{i} - t_{1}) = r_{i}(x_k) - r_{1}(x_k),
\]

where \(c\) is speed of light and \(i = 1, 2, 3, ..., M\). Let \(\dot{r}_{i}\) be the true range rate between the source and receiver \(i\)

\[
\dot{r}_i = \frac{(\hat{u}_k - \hat{s}_i)^T (u_k - s_i)}{||u_k - s_i||},
\]

where \(\hat{u}_k\) is velocity vector of the target. The FDoA between receiver \(i\) and receiver \(1\) is

\[
\dot{r}_{i,1} = \frac{(\hat{u}_k - \hat{s}_i)^T (u_k - s_i)}{||u_k - s_i||} - \frac{(\hat{u}_k - \hat{s}_i)^T (u_k - s_i)}{||u_k - s_i||}.
\]
The TDoA/FDoA measurements between $s_1$ and $s_i$ are stacked to yield a full measurement vector $Z_k$ at time $t_k$:

$$Z_k = H(x_k) + V_k,$$  \hspace{1cm} (11)

where $H(x_k) = [h_{21}(x_k) \ h_{31}(x_k) \ldots h_{M1}(x_k)]$, and the $V_k$ is white Gaussian noise

$$h_{ji}(x_k) = \begin{bmatrix} r_{1i}(x_k) \\ r_{2i}(x_k) \end{bmatrix},$$  \hspace{1cm} (12)

where $i = 2, \ldots, M$.

### III. PARTICLE FILTER

The particle filters are sequential Monte Carlo methods based on point mass representations of probability densities, which can be applied to any state-space model and which generalize the traditional Kalman filtering methods [3].

#### Algorithm 1: Target Tracking TDoA/FDoA based on Particle Filter

1. Initialize the state $x_0$
2. Generate $N$ particles $x_{0j} \sim N(x_0, \sigma^2 I_0)$
3. for $j = 1, 2, \ldots, N$ do
4. \hspace{0.5cm} Set initial weights $w_{0j} = 1/N$
5. \hspace{0.5cm} for $k = 1, 2, \ldots$ do
6. \hspace{1cm} $x_{kj} = \Phi x_{k-1j} + \Gamma w_{k-1j}$
7. \hspace{1cm} where, $w_{k-1j} \sim N(0, Q_w)$
8. \hspace{1cm} $w_{kj} = p(Z_k|x_{kj})$
9. \hspace{1cm} where, $p(Z_k|x_{kj}) \sim N(h(x_{kj}), Q_v)$
10. \hspace{1cm} Normalize weight $\bar{w}_{kj} = w_{kj}/\sum_{j=1}^N \bar{w}_{kj}$
11. \hspace{1cm} $\hat{x} = \sum_{j=1}^N \bar{w}_{kj} x_{kj}$
12. \hspace{1cm} Resampling
13. end for
14. end for

In here, $Q_v = \text{diag}[\sigma_v, \sigma_\nu]$ is covariance matrix of TDoA/FDoA measurements noise. The core idea of the sampling importance resampling (SIR) Particle filter is resampled in a way that further increases or decreases the number of particles in proportion to the weight of each particle. In addition, it has the effect of reducing the degradation phenomenon and calculation time by resampling in each repetition process.

### IV. SIMULATION AND RESULT

In this paper, the simulation experiment environment configured as follows. The moving target starts at $[10000, 10000, 5000]^\top$ [m] and moves to velocity $v = [-30, -30, -30]^\top$ [m/s] and accelerometer $a = [0.1, 0.1, 0.1]^\top$ [m/s\(^2\)]. Also, 4 receivers are dispersed as follows, $s_1 = [0, 20000, 0]^\top$, $s_2 = [20000, 0, 0]^\top$, $s_3 = [20000, 20000, 1500]^\top$, $s_4 = [0, 0, 3000]^\top$ with unit [m], as shown in Fig. 1. The reference sensor is $s_1$, and the particle filter parameter was applied the parameter as shown in Table I. The performance of the proposed localization method was analyzed through 100 Monte Carlo trials using MATLAB, and Fig. 1 is the result of showing the true trajectory and estimated trajectory for one of the Monte Carlo trials. Through the proposed method, we can see that object movement tracking was possible along the actual location in a noisy environment, and the average RMSE was verified to be 66 [m].

### V. CONCLUSION

In this paper, we proposed a localization algorithm using TDoA/FDoA measurements based on particle filter. In addition, the performance of the proposed algorithm was verified through MATLAB simulation. As a result, it was appeared that the moving object was well tracked through the proposed algorithm even in an environment containing noise in 3D space.

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