

Forward Secure Message Franking

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Abstract. Message franking is introduced by Facebook in end-to-end encrypted messaging services. It allows to produce verifiable reports of malicious messages by including cryptographic proofs generated by Facebook. Recently, Grubbs et al. (CRYPTO'17) proceeded with the formal study of message franking and introduced committing authenticated encryption with associated data (CAEAD) as a core primitive for obtaining message franking.

In this work, we aim to enhance the security of message franking and propose forward security for message franking. It guarantees the security associated with the past keys even if the current keys are exposed. Firstly, we propose the notion of key-evolving message franking including additional key update algorithms. Then, we formalize forward security for five security requirements: confidentiality, ciphertext integrity, unforgeability, receiver binding, and sender binding. Finally, we show a construction of forward secure message franking based on CAEAD, forward secure pseudorandom generator, and forward secure message authentication code.

Keywords: message franking · forward security · abusive verifiable reports

1 Introduction

1.1 Background

Message Franking. Billions of people use messaging services such as WhatsApp [22], Signal [19], and Facebook Messenger [11]. In these services, the security goal is end-to-end security: the third party including the service providers cannot compromise the security of messages. Keeping the messages secret from the service providers has recently led to the following problem: when the receiver receives malicious messages such as spam, phishing links, and so on, he/she attempts to report them to the service providers so that they could take measures against the sender. However, the service providers are not able to judge the reported messages were actually sent by the particular sender.

To tackle this problem, Facebook introduced the notion of message franking [12]. In message franking, the sender generates a commitment to a message,

called a franking tag, along with a ciphertext generated using a secret key (which is shared with the receiver) and sends them to Facebook. Next, Facebook generates a cryptographic proof, called a reporting tag, from the franking tag using a tagging key (which is held by Facebook) and gives the ciphertext, the franking tag, and the reporting tag to the receiver. Then, the receiver decrypts the ciphertext using the secret key, gets the message and an opening of the franking tag, and verifies the validity of the franking tag. If the receiver wants to report a malicious message, he/she sends the reporting tag and the opening to Facebook in addition to the message. It enables Facebook to verify the specific sender actually sent the reported messages by validating the reporting tag using the tagging key.

Grubbs, Lu, and Ristenpart [13] initiated the formal study of message franking. They introduced committing authenticated encryption with associated data (CAEAD) as a core primitive for message franking. CAEAD is authenticated encryption with associated data (AEAD) that uses a small part of the ciphertext as a commitment to the message.

Forward Security. In general, most of cryptographic primitives are designed to be secure as long as their secret keys are not compromised. Thus, the exposure of secret keys is the greatest threat for many cryptographic schemes. Especially, if secret keys are compromised, the security of schemes is at risk not only after compromising but also in the past.

Forward security [9, 14] is one of the major solutions to address the exposure of secret keys, which ensures that compromise of the secret keys in the present does not influence the security of ciphertexts and tags generated in the past. Roughly, considering forward security, the lifetime of the system is divided into n time periods and secret keys are updated in each period so that any past secret keys cannot be calculated from the current secret keys. To date, forward security was defined in a digital signature scheme [3], a symmetric encryption scheme [4], and an asymmetric encryption scheme [5]. Recently, the definition of forward security has also been considered for practical cryptographic primitives, such as a non-interactive key exchange scheme [20], a 0-RTT key exchange protocol [8, 15], a 0-RTT session resumption protocol [2], and Signal protocol [1, 7].

In a message franking scheme, the exposure of secret keys causes malicious adversaries to decrypt and tamper with the past ciphertexts and forge the past reporting tags. Moreover, it also enables them to falsely report messages that were actually not sent and generate messages that fail verification. To avoid these problems, the challenge of achieving forward security in a message franking scheme is important.

1.2 Our Contribution

In this paper, we initiate the study on forward security for message franking. For capturing forward security, we firstly formalize key-evolving message franking, which includes two key update algorithms for a secret key and a tagging key, respectively.

Roughly, forward security in message franking guarantees the security of the ciphertexts and reporting tags generated with the past keys, even if the current keys are exposed. More precisely, we define forward security on key-evolving message franking for five security requirements: confidentiality, ciphertext integrity, unforgeability, receiver binding, and sender binding. Confidentiality and ciphertext integrity are the security notions for ciphertexts, while unforgeability, receiver binding, and sender binding are the security notions for reporting tags. Confidentiality guarantees that the information about the messages is not leaked from the ciphertexts and ciphertext integrity guarantees that the ciphertexts cannot be tampered with. Similar to the previous work on message franking [13], we adapt multiple-opening (MO) security for confidentiality and ciphertext integrity. MO security allows to securely encrypt multiple different messages under the same secret key. Unforgeability guarantees that reporting tags are not forged, receiver binding guarantees that the receiver is not able to report messages that were not actually sent, and sender binding guarantees that the sender is not able to send malicious messages that cannot be reported. See Section 3.1 for the details.

We show a construction of a key-evolving message franking scheme combining a CAEAD scheme with a forward secure pseudorandom generator (PRG) and a forward secure message authentication code (MAC) scheme. In a nutshell, we use a CAEAD scheme to generate ciphertexts and franking tags and decrypt them with a secret key updated by a forward secure PRG. Moreover, we use a forward secure MAC scheme to generate and verify reporting tags with an updated tagging key. See Section 4.2 for the details.

1.3 Related Work

As mentioned above, Grubbs et al. [13] introduced CAEAD as a core primitive for message franking. They provided two constructions of a CAEAD scheme: Commit-then-Encrypt (CtE) that combines a commitment scheme with an authenticated encryption with associated data (AEAD) scheme and Committing Encrypt-and-PRF (CEP) that uses a nonce-based PRG, a pseudorandom function (PRF), and a collision resistant PRF.

Dodis, Grubbs, Ristenpart, and Woodage [10] showed the attack against message franking for attachments. They also introduced encryptment, which is simplified CAEAD for design and analyse and provided an encryptment scheme using a hash function. Hirose [16] provided an encryptment scheme using a tweakable block cipher (TBC).

Leontiadis and Vaudenay [18] proposed a new security definition, called multiple-opening indistinguishability with partial opening (MO-IND-PO), which ensures confidentiality of unreported parts of the messages after reporting malicious parts. Chen and Tang [6] introduced targeted opening committing authenticated encryption with associated data (TOCE), which allows the receiver to report only the abusive parts of the messages for verification.

Huguenin-Dumittan and Leontiadis [17] introduced message franking channel, which is resistant to replay attacks, out-of-order delivery and message drops.

They provided a construction of a message franking channel using a CAEAD scheme and a MAC scheme.

Tyagi, Grubbs, Len, Miers, and Ristenpart [21] introduced asymmetric message franking (AMF) to achieve content moderation under the condition that the sender and receiver identities are hidden from the service providers. They provided a construction of an AMF scheme using an applied technique of a designated verifier signature scheme.

2 Preliminaries

2.1 Notation

For a positive integer n , we write $[n]$ to denote the set $\{1, \dots, n\}$. For a finite set X , we write $x \xleftarrow{\$} X$ to denote sampling x from X uniformly at random and $|X|$ to denote the cardinality of X . For a string x , we write $|x|$ to denote the length of x . For an algorithm \mathcal{A} , we write $y \leftarrow \mathcal{A}(x)$ to denote running \mathcal{A} on the input x to produce the output y . λ denotes a security parameter. A function $f(\lambda)$ is a negligible function if $f(\lambda)$ tends to 0 faster than $\frac{1}{\lambda^c}$ for every constant $c > 0$. $\text{negl}(\lambda)$ denotes an unspecified negligible function.

2.2 Forward Secure Pseudorandom Generator

Definition 1 (Stateful Pseudorandom Generator [4]). A stateful pseudorandom generator $\text{sPRG} = (\text{Key}, \text{Next})$ is a tuple of two algorithms associated with a state space \mathcal{ST} and a block space \mathcal{OUT} defined as follows.

- $St_0 \leftarrow \text{Key}(1^\lambda, n)$: The key generation algorithm Key takes as input a security parameter 1^λ and the total number of time periods n and outputs the initial state St_0 .
- $(Out_i, St_i) \leftarrow \text{Next}(St_{i-1})$: The next step algorithm Next takes as input the current state St_{i-1} and outputs a output block Out_i and the next state St_i .

Definition 2 (Forward Security). Let $n \geq 1$ be any integer. For a stateful pseudorandom generator $\text{sPRG} = (\text{Key}, \text{Next})$, we define the forward security game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.

1. \mathcal{CH} generates $St_0 \leftarrow \text{Key}(1^\lambda, n)$, sets $i := 0$, and chooses $b \xleftarrow{\$} \{0, 1\}$.
2. \mathcal{CH} sets $i := i + 1$. Depending on the value of b , \mathcal{CH} proceeds as follows.
 - If $b = 1$, \mathcal{CH} computes $(Out_i, St_i) \leftarrow \text{Next}(St_{i-1})$ and sends Out_i to \mathcal{A} .
 - If $b = 0$, \mathcal{CH} computes $(Out'_i, St_i) \leftarrow \text{Next}(St_{i-1})$ and $Out_i \xleftarrow{\$} \{0, 1\}^\lambda$ sends Out_i to \mathcal{A} .
3. \mathcal{A} outputs $d \in \{0, 1\}$.
4. If $d = 1$ or $i = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 and 3.
5. \mathcal{CH} sends St_i to \mathcal{A} .
6. \mathcal{A} outputs $b' \in \{0, 1\}$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{sPRG}, \mathcal{A}}^{\text{FS-PRG}}(\lambda) := \left| \Pr[b' = b] - \frac{1}{2} \right|.$$

We say that sPRG is forward secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{sPRG}, \mathcal{A}}^{\text{FS-PRG}}(\lambda) = \text{negl}(\lambda)$.

2.3 Forward Secure Message Authentication Code

Definition 3 (Key-Evolving Message Authentication Code [4]). A key-evolving message authentication code scheme $\text{FSMAC} = (\text{Gen}, \text{Upd}, \text{Tag}, \text{Ver})$ is a tuple of four algorithms associated with a key space \mathcal{K} , a message space \mathcal{M} , and a tag space \mathcal{T} defined as follows.

- $K_0 \leftarrow \text{Gen}(1^\lambda, n)$: The key generation algorithm Gen takes as input a security parameter 1^λ and the total number of time period n and outputs the initial secret key K_0 .
- $K_i \leftarrow \text{Upd}(K_{i-1})$: The key update algorithm Upd takes as input the current secret key K_{i-1} and outputs the next secret key K_i .
- $(\tau, i) \leftarrow \text{Tag}(K_i, M)$: The tagging algorithm Tag takes as input the current secret key K_i and a message M and outputs a tag τ and the current time period i .
- $b \leftarrow \text{Ver}(K_i, M, (\tau, \hat{i}))$: The verification algorithm Ver takes as input the current tagging key K_i , a message M , and a pair of a tag and a time period (τ, \hat{i}) and outputs a bit b , with 1 meaning accept and 0 meaning reject.

As the correctness, we require that for any $n, \lambda \in \mathbb{N}$, $M \in \mathcal{M}$, $K_0 \leftarrow \text{Gen}(1^\lambda, n)$, and $K_i \leftarrow \text{Upd}(K_{i-1})$ for $i = 1, \dots, n$, $\text{Ver}(K_i, M, \text{Tag}(K_i, M)) = 1$ holds for all $\hat{i} \in [n]$.

Definition 4 (FS-sEUF-CMA Security). Let $n \geq 1$ be some integer. For a key-evolving MAC scheme FSMAC , we define the forward secure strong existentially unforgeability under adaptive chosen message attack (FS-sEUF-CMA security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.

1. \mathcal{CH} generates $K_0 \leftarrow \text{Gen}(1^\lambda, n)$ and sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{CH} sets $i := i + 1$ and computes $K_i \leftarrow \text{Upd}(K_{i-1})$.
3. \mathcal{A} is allowed to make tagging queries. On tagging queries M , \mathcal{CH} computes $(\tau, i) \leftarrow \text{Tag}(K_i, M)$, gives (τ, i) to \mathcal{A} , and appends $(M, (\tau, i))$ to S_i .
4. \mathcal{A} outputs $d \in \{0, 1\}$.
5. If $d = 1$ or $i = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} sends K_i to \mathcal{A} .
7. \mathcal{A} outputs $(M^*, (\tau^*, i^*))$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMAC}, \mathcal{A}}^{\text{FS-sEUF-CMA}}(\lambda) := \Pr[\text{Ver}(K_{i^*}, M^*, (\tau^*, i^*)) = 1 \wedge (M^*, (\tau^*, i^*)) \notin S_{i^*} \wedge 1 \leq i^* < i].$$

We say that FSMAC is FS-sEUF-CMA secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMAC}, \mathcal{A}}^{\text{FS-sEUF-CMA}}(\lambda) = \text{negl}(\lambda)$.

2.4 Committing Authenticated Encryption with Associated Data

Definition 5 (Committing Authenticated Encryption with Associated Data [13]). A committing authenticated encryption with associated data (CAEAD) scheme $\text{CAEAD} = (\text{Gen}, \text{Enc}, \text{Dec}, \text{Ver})$ is a tuple of four algorithms associated with a key space \mathcal{K} , a header space \mathcal{H} , a message space \mathcal{M} , an opening space \mathcal{O} , a ciphertext space \mathcal{C} , and a franking tag space \mathcal{B} defined as follows.

- $K \leftarrow \text{Gen}(1^\lambda)$: The key generation algorithm Gen takes as input a security parameter 1^λ and outputs a secret key K .
- $(C_1, C_2) \leftarrow \text{Enc}(K, H, M)$: The encryption algorithm Enc takes as input a secret key K , a header H , and a message M and outputs a ciphertext C_1 and a franking tag C_2 .
- $(M', O) \leftarrow \text{Dec}(K, H, C_1, C_2)$: The decryption algorithm Dec takes as input a secret key K , a header H , a ciphertext C_1 , and a franking tag C_2 and outputs a message M' and an opening O .
- $b \leftarrow \text{Ver}(H, M, O, C_2)$: The verification algorithm Ver takes as input a header H , a message M , an opening O , and a franking tag C_2 and outputs a bit b , with 1 meaning accept and 0 meaning reject.

As the correctness, we require that for any $n, \lambda \in \mathbb{N}$, $M \in \mathcal{M}$, $H \in \mathcal{H}$, $K \leftarrow \text{Gen}(1^\lambda)$, $(C_1, C_2) \leftarrow \text{Enc}(K, H, M)$, and $(M', O) \leftarrow \text{Dec}(K, H, C_1, C_2)$, $M' = M$ and $\text{Ver}(H, M', O, C_2) = 1$ holds.

Definition 6 (MO-IND Security). For a CAEAD scheme CAEAD , we define the multiple-opening indistinguishability (MO-IND security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.

1. \mathcal{CH} generates $K \leftarrow \text{Gen}(1^\lambda)$ and sets $S := \emptyset$.
2. \mathcal{A} is allowed to make encryption queries and decryption queries as follows.
 - On encryption queries of the form (H, M) , \mathcal{CH} computes $(C_1, C_2) \leftarrow \text{Enc}(K, H, M)$, gives (C_1, C_2) to \mathcal{A} , and appends (H, C_1, C_2) to S .
 - On decryption queries of the form (H, C_1, C_2) , if $(H, C_1, C_2) \notin S$, \mathcal{CH} gives \perp to \mathcal{A} . Otherwise, \mathcal{CH} computes $(M', O) \leftarrow \text{Dec}(K, H, C_1, C_2)$ and gives (M', O) to \mathcal{A} .
3. \mathcal{A} sends (H^*, M_0^*, M_1^*) to \mathcal{CH} , where $|M_0^*| = |M_1^*|$.
4. \mathcal{CH} chooses a challenge bit $b \xleftarrow{\$} \{0, 1\}$, computes $(C_1^*, C_2^*) \leftarrow \text{Enc}(K, H^*, M_b^*)$ and sends (C_1^*, C_2^*) to \mathcal{A} .
5. \mathcal{A} is allowed to make the same encryption and decryption queries as with Step 2 except that (H^*, C_1^*, C_2^*) cannot be queried in the decryption query.
6. \mathcal{A} outputs $b' \in \{0, 1\}$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{CAEAD}, \mathcal{A}}^{\text{MO-IND}}(\lambda) := \left| \Pr[b' = b] - \frac{1}{2} \right|.$$

We say that CAEAD is MO-IND secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{CAEAD}, \mathcal{A}}^{\text{MO-IND}}(\lambda) = \text{negl}(\lambda)$.

Remark 1. While Grubbs et al. [13] used the definition which guarantees that ciphertexts cannot be distinguished from random strings, our definition ensures that, for the two messages outputted by the adversary, it is hard to identify which message the received ciphertext is generated from. Similar to [6], we can construct a CAEAD scheme satisfying MO-IND security from an authenticated encryption with associated data (AEAD) scheme and a commitment scheme.

Definition 7 (MO-CTXT Security). *For a CAEAD scheme CAEAD, we define the multiple-opening ciphertext integrity (MO-CTXT security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.*

1. \mathcal{CH} generates $K \leftarrow \text{Gen}(1^\lambda)$ and sets $S := \emptyset$.
2. \mathcal{A} is allowed to make encryption queries and decryption queries as follows.
 - On encryption queries of the form (H, M) , \mathcal{CH} computes $(C_1, C_2) \leftarrow \text{Enc}(K, H, M)$, gives (C_1, C_2) to \mathcal{A} , and appends (H, C_1, C_2) to S .
 - On decryption queries of the form (H, C_1, C_2) , \mathcal{CH} computes $(M', O) \leftarrow \text{Dec}(K, H, C_1, C_2)$ and gives (M', O) to \mathcal{A} .
3. \mathcal{A} outputs (H^*, C_1^*, C_2^*) .

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{CAEAD}, \mathcal{A}}^{\text{MO-CTXT}}(\lambda) := \Pr[M^* \neq \perp \wedge (H^*, C_1^*, C_2^*) \notin S : (M^*, O^*) \leftarrow \text{Dec}(K, H^*, C_1^*, C_2^*)].$$

We say that CAEAD is MO-CTXT secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{CAEAD}, \mathcal{A}}^{\text{MO-CTXT}}(\lambda) = \text{negl}(\lambda)$.

Definition 8 (R-BIND Security). *We say that a CAEAD scheme CAEAD satisfies the receiver binding (R-BIND security) if for any $(H, M) \neq (H', M')$, $\text{Ver}(H, M, O, C_2^*) = \text{Ver}(H', M', O', C_2^*) = 1$ never holds.*

Definition 9 (S-BIND Security). *We say that a CAEAD scheme CAEAD satisfies the sender binding (S-BIND security) if for any $K \in \mathcal{K}$, $H \in \mathcal{H}$, $C_1 \in \mathcal{C}$, $C_2 \in \mathcal{B}$, and $(M', O) \leftarrow \text{Dec}(K, H, C_1, C_2)$, $\text{Ver}(H, M', O, C_2) = 0$ and $M' \neq \perp$ never holds.*

3 Forward Secure Message Franking

In this section, we introduce forward secure message franking. First, in Section 3.1, we define the syntax and its correctness of key-evolving message franking. Then, in Section 3.2, we provide forward security definitions for key-evolving message franking.

3.1 Syntax

In this section, we provide the syntax of a key-evolving message franking scheme. A key-evolving message franking scheme includes two additional algorithms for updating a secret key and a tagging key asynchronously.

Definition 10 (Key-Evolving Message Franking). *A key-evolving message franking scheme FSMF is a tuple of eight algorithms (SKGen, TKGen, SKUpd, TKUpd, Enc, Tag, Dec, Ver) associated with a secret key space SK , a tagging key space TK , a header space \mathcal{H} , a message space \mathcal{M} , an opening space \mathcal{O} , a ciphertext space \mathcal{C} , a franking tag space \mathcal{B} , and a reporting tag space \mathcal{T} defined as follow.*

- $SK_0 \leftarrow SKGen(1^\lambda, n)$: The secret key generation algorithm SKGen takes as input a security parameter 1^λ and the total number of time periods n and outputs the initial secret key SK_0 .
- $TK_0 \leftarrow TKGen(1^\lambda, n)$: The tagging key generation algorithm TKGen takes as input a security parameter 1^λ and the total number of time periods n and outputs the initial tagging key TK_0 .
- $SK_i \leftarrow SKUpd(SK_{i-1})$: The secret key update algorithm SKUpd takes as input the current secret key SK_{i-1} and outputs the next secret key SK_i .
- $TK_j \leftarrow TKUpd(TK_{j-1})$: The tagging key update algorithm TKUpd takes as input the current tagging key TK_{j-1} and outputs the next tagging key TK_j .
- $(C_1, C_2, i) \leftarrow Enc(SK_i, H, M)$: The encryption algorithm Enc takes as input the current secret key SK_i , a header H , and a message M and outputs a ciphertext C_1 , a franking tag C_2 , and the current time period i .
- $(\tau, j) \leftarrow Tag(TK_j, C_2)$: The tagging algorithm Tag takes as input the current tagging key TK_j and a franking tag C_2 and outputs a reporting tag τ and the current time period j .
- $(M', O) \leftarrow Dec(SK_i, H, (C_1, C_2, \hat{i}))$: The decryption algorithm Dec takes as input the current secret key SK_i , a header H , and a tuple of a ciphertext, a franking tag, and a time period (C_1, C_2, \hat{i}) and outputs a message M' and an opening O .
- $b \leftarrow Ver(TK_j, H, M, O, C_2, (\tau, \hat{j}))$: The verification algorithm Ver takes as input the current tagging key TK_j , a header H , a message M , an opening O , a franking tag C_2 , and a pair of a reporting tag and a time period (τ, \hat{j}) and outputs a bit b , with 1 meaning accept and 0 meaning reject.

As the correctness, we require that for any $n, \lambda \in \mathbb{N}$, $M \in \mathcal{M}$, $H \in \mathcal{H}$, $SK_0 \leftarrow SKGen(1^\lambda, n)$, $TK_0 \leftarrow TKGen(1^\lambda, n)$, $SK_i \leftarrow SKUpd(SK_{i-1})$, $TK_j \leftarrow TKUpd(TK_{j-1})$ for $i, j = 1, \dots, n$, $(C_1, C_2, \hat{i}) \leftarrow Enc(SK_i, H, M)$, $(\tau, \hat{j}) \leftarrow Tag(TK_{\hat{j}}, C_2)$, and $(M', O) \leftarrow Dec(SK_{\hat{i}}, H, (C_1, C_2, \hat{i}))$, $M' = M$ and $Ver(TK_{\hat{j}}, H, M', O, C_2, (\tau, \hat{j})) = 1$ holds for all $\hat{i}, \hat{j} \in [n]$.

3.2 Security Definitions

In this section, we define forward security for five security notions: confidentiality, ciphertext integrity, unforgeability, receiver binding, and sender binding.

Confidentiality. Intuitively, confidentiality guarantees that the information of the messages is not leaked from the corresponding ciphertexts. More formally, we require that adversaries with the information of the current secret key cannot distinguish ciphertexts generated by the past secret key. We apply multiple-opening (MO) security [13] to confidentiality. MO security ensures that multiple ciphertexts, encrypted under the same secret key, whose opening is known do not endanger the security of other ciphertexts whose opening is not known. In MO security, the adversaries can make decryption queries in addition to encryption queries to learn the openings of the ciphertexts generated via encryption queries.

Definition 11 (FS-MO-IND Security). *Let $n \geq 1$ be some integer. For a key-evolving message franking scheme FSMF, we define the forward secure multiple-opening indistinguishability (FS-MO-IND security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.*

1. \mathcal{CH} generates $\text{SK}_0 \leftarrow \text{SKGen}(1^\lambda, n)$ and sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{CH} sets $i := i + 1$ and computes $\text{SK}_i \leftarrow \text{SKUpd}(\text{SK}_{i-1})$.
3. \mathcal{A} is allowed to make encryption queries and decryption queries as follows.
 - On encryption queries of the form (H, M) , \mathcal{CH} computes $(C_1, C_2, i) \leftarrow \text{Enc}(\text{SK}_i, H, M)$, gives (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
 - On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $(H, (C_1, C_2, \hat{i})) \notin S_i$, \mathcal{CH} gives \perp to \mathcal{A} . Otherwise, \mathcal{CH} computes $(M', O) \leftarrow \text{Dec}(\text{SK}_i, H, (C_1, C_2, \hat{i}))$ and gives (M', O) to \mathcal{A} .
4. \mathcal{A} sends $(d, H^*, M_0^*, M_1^*, i^*)$ to \mathcal{CH} , where $|M_0^*| = |M_1^*|$.
5. If $d = 1$ or $i = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} chooses a challenge bit $b \xleftarrow{\$} \{0, 1\}$. If $i^* \geq i$, \mathcal{CH} chooses $b' \xleftarrow{\$} \{0, 1\}$ and terminates. Otherwise, \mathcal{CH} computes $(C_1^*, C_2^*, i^*) \leftarrow \text{Enc}(\text{SK}_{i^*}, H^*, M_b^*)$ and sends $(\text{SK}_i, (C_1^*, C_2^*, i^*))$ to \mathcal{A} .
7. \mathcal{A} outputs $b' \in \{0, 1\}$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-IND}}(\lambda) := \left| \Pr[b' = b] - \frac{1}{2} \right|.$$

We say that FSMF is FS-MO-IND secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-IND}}(\lambda) = \text{negl}(\lambda)$.

Ciphertext Integrity. Intuitively, ciphertext integrity guarantees that ciphertexts are not tampered with. More formally, we require that adversaries with the information of the current secret key cannot generate a new ciphertext which is correctly decrypted by the past secret key. Similar to the above confidentiality, we apply the MO security to ciphertext integrity.

Definition 12 (FS-MO-CTXT Security). *Let $n \geq 1$ be some integer. For a key-evolving message franking scheme FSMF, we define the forward secure multiple-opening ciphertext integrity (FS-MO-CTXT security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.*

1. \mathcal{CH} generates $\text{SK}_0 \leftarrow \text{SKGen}(1^\lambda, n)$ and sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{CH} sets $i := i + 1$ and computes $\text{SK}_i \leftarrow \text{SKUpd}(\text{SK}_{i-1})$.
3. \mathcal{A} is allowed to make encryption queries and decryption queries as follows.
 - On encryption queries of the form (H, M) , \mathcal{CH} computes $(C_1, C_2, i) \leftarrow \text{Enc}(\text{SK}_i, H, M)$, gives (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
 - On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, \mathcal{CH} computes $(M', O) \leftarrow \text{Dec}(\text{SK}_i, H, (C_1, C_2, \hat{i}))$ and gives (M', O) to \mathcal{A} .
4. \mathcal{A} outputs $d \in \{0, 1\}$.
5. If $d = 1$ or $i = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} sends SK_i to \mathcal{A} .
7. \mathcal{A} outputs $(H^*, (C_1^*, C_2^*, i^*))$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-CTXT}}(\lambda) := \Pr[M^* \neq \perp \wedge (H^*, (C_1^*, C_2^*, i^*)) \notin S_{i^*} \wedge 1 \leq i^* < i : (M^*, O^*) \leftarrow \text{Dec}(\text{SK}_{i^*}, H^*, (C_1^*, C_2^*, i^*))].$$

We say that FSMF is FS-MO-CTXT secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-CTXT}}(\lambda) = \text{negl}(\lambda)$.

Unforgeability. Intuitively, unforgeability guarantees that reporting tags are not forged. More formally, we require that adversaries with the information of the current tagging key cannot generate a new reporting tag which is successfully verified by the past tagging key.

Definition 13 (FS-UNF Security). Let $n \geq 1$ be some integer. For a key-evolving message franking scheme FSMF, we define the forward secure unforgeability (FS-UNF security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.

1. \mathcal{CH} generates $\text{TK}_0 \leftarrow \text{TKGen}(1^\lambda, n)$ and sets $j := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{CH} sets $j := j + 1$ and computes $\text{TK}_j \leftarrow \text{TKUpd}(\text{TK}_{j-1})$.
3. \mathcal{A} is allowed to make tagging queries. On tagging queries C_2 , \mathcal{CH} computes $(\tau, j) \leftarrow \text{Tag}(\text{TK}_j, C_2)$, gives (τ, j) to \mathcal{A} , and appends $(C_2, (\tau, j))$ to S_j .
4. \mathcal{A} outputs $d \in \{0, 1\}$.
5. If $d = 1$ or $j = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} sends TK_j to \mathcal{A} .
7. \mathcal{A} outputs $(H^*, M^*, O^*, C_2^*, (\tau^*, j^*))$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-UNF}}(\lambda) := \Pr[\text{Ver}(\text{TK}_{j^*}, H^*, M^*, O^*, C_2^*, (\tau^*, j^*)) = 1 \wedge (C_2^*, (\tau^*, j^*)) \notin S_{j^*} \wedge 1 \leq j^* < j].$$

We say that FSMF is FS-UNF secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-UNF}}(\lambda) = \text{negl}(\lambda)$.

Receiver Binding. Intuitively, receiver binding guarantees that the receiver is not able to report messages which were not actually sent. More formally, we require that adversaries with the information of the current tagging key cannot generate two messages successfully verified by the past tagging key for a pair of franking tag and reporting tag.

Definition 14 (FS-R-BIND Security). *Let $n \geq 1$ be some integer. For a key-evolving message franking scheme FSMF, we define the forward secure receiver binding (FS-R-BIND security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.*

1. \mathcal{CH} generates $\text{TK}_0 \leftarrow \text{TKGen}(1^\lambda, n)$ and sets $j := 0$.
2. \mathcal{CH} sets $j := j + 1$ and computes $\text{TK}_j \leftarrow \text{TKUpd}(\text{TK}_{j-1})$.
3. \mathcal{A} is allowed to make tagging queries. On tagging queries of C_2 , \mathcal{CH} computes $(\tau, j) \leftarrow \text{Tag}(\text{TK}_j, C_2)$ and gives (τ, j) to \mathcal{A} .
4. \mathcal{A} outputs $d \in \{0, 1\}$.
5. If $d = 1$ or $j = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} sends TK_j to \mathcal{A} .
7. \mathcal{A} outputs $((H, M, O), (H', M', O'), C_2^*, (\tau^*, j^*))$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-R-BIND}}(\lambda) := \Pr[\text{Ver}(\text{TK}_{j^*}, H, M, O, C_2^*, (\tau^*, j^*)) = \text{Ver}(\text{TK}_{j^*}, H', M', O', C_2^*, (\tau^*, j^*)) = 1 \wedge (H, M) \neq (H', M') \wedge 1 \leq j^* < j].$$

We say that FSMF is FS-R-BIND secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-R-BIND}}(\lambda) = \text{negl}(\lambda)$.

Sender Binding. Intuitively, sender binding guarantees that the sender is not able to send malicious messages that cannot be reported. More formally, we require that adversaries with the information of the current tagging key cannot generate a ciphertext that are correctly decrypted but fail to verify by the past tagging key.

Definition 15 (FS-S-BIND Security). *Let $n \geq 1$ be some integer. For a key-evolving message franking scheme FSMF, we define the forward secure sender binding (FS-S-BIND security) game between a challenger \mathcal{CH} and an adversary \mathcal{A} as follows.*

1. \mathcal{CH} generates $\text{TK}_0 \leftarrow \text{TKGen}(1^\lambda, n)$ and sets $j := 0$.
2. \mathcal{CH} sets $j := j + 1$ and computes $\text{TK}_j \leftarrow \text{TKUpd}(\text{TK}_{j-1})$.
3. \mathcal{A} is allowed to make tagging queries. On tagging queries of C_2 , \mathcal{CH} computes $(\tau, j) \leftarrow \text{Tag}(\text{TK}_j, C_2)$ and gives (τ, j) to \mathcal{A} .
4. \mathcal{A} outputs $d \in \{0, 1\}$.
5. If $d = 1$ or $j = n$, \mathcal{CH} proceeds to the next Step, else repeats Steps 2 through 4.
6. \mathcal{CH} sends TK_j to \mathcal{A} .
7. \mathcal{A} outputs $(j^*, \text{SK}_i, H, (C_1, C_2, \hat{i}))$.

$\text{SKGen}(1^\lambda, n)$ $St_0 \leftarrow \text{sPRG.Key}(1^\lambda, n)$ Output $\text{SK}_0 = (0, \epsilon, St_0)$	$\text{TKGen}(1^\lambda, n)$ $\text{TK}_0 \leftarrow \text{FSMAC.Gen}(1^\lambda, n)$ Output TK_0
$\text{SKUpd}(\text{SK}_{i-1})$ Parse $\text{SK}_{i-1} = (i-1, \text{Out}_{i-1}, St_{i-1})$ $(\text{Out}_i, St_i) \leftarrow \text{sPRG.Next}(St_{i-1})$ Output $\text{SK}_i = (i, \text{Out}_i, St_i)$	$\text{TKUpd}(\text{TK}_{j-1})$ $\text{TK}_j \leftarrow \text{FSMAC.Upd}(\text{TK}_{j-1})$ Output TK_j
$\text{Enc}(\text{SK}_i, H, M)$ Parse $\text{SK}_i = (i, \text{Out}_i, St_i)$ $(C_1, C_2) \leftarrow \text{CAEAD.Enc}(\text{Out}_i, H, M)$ Output (C_1, C_2, i)	$\text{Tag}(\text{TK}_j, C_2)$ $(\tau, j) \leftarrow \text{FSMAC.Tag}(\text{TK}_j, C_2)$ Output (τ, j)
$\text{Dec}(\text{SK}_i, H, (C_1, C_2, \hat{i}))$ Parse $\text{SK}_i = (i, \text{Out}_i, St_i)$ If $\hat{i} \neq i$, Output \perp else $(M', O) \leftarrow \text{CAEAD.Dec}(\text{Out}_i, H, C_1, C_2)$ Output (M', O)	$\text{Ver}(\text{TK}_j, H, M, O, C_2, (\tau, \hat{j}))$ If $\text{CAEAD.Ver}(H, M, O, C_2) = 0$ Output 0 else $b \leftarrow \text{FSMAC.Ver}(\text{TK}_j, C_2, (\tau, \hat{j}))$ Output b

Fig. 1. Construction of key-evolving message franking scheme.

8. \mathcal{CH} computes $(\tau, j^*) \leftarrow \text{Tag}(\text{TK}_{j^*}, C_2)$ and $(M', O) \leftarrow \text{Dec}(\text{SK}_i, H, (C_1, C_2, \hat{i}))$.

In this game, we define the advantage of the adversary \mathcal{A} as

$$\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-S-BIND}}(\lambda) := \Pr[\text{Ver}(\text{TK}_{j^*}, H, M', O, C_2, (\tau, j^*)) = 0 \wedge M' \neq \perp \wedge 1 \leq j^* < j].$$

We say that FSMF is FS-S-BIND secure if for any PPT adversary \mathcal{A} , we have $\text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-S-BIND}}(\lambda) = \text{negl}(\lambda)$.

4 Construction of Key-Evolving Message Franking

In this section, we show our construction of a key-evolving message franking scheme. First, in Section 4.1, we provide the formal description of our construction. Then, in Section 4.2, we give security proofs for our construction.

4.1 Construction

Let $\text{sPRG} = (\text{sPRG.Key}, \text{sPRG.Next})$ be a stateful generator, $\text{FSMAC} = (\text{FSMAC.Gen}, \text{FSMAC.Upd}, \text{FSMAC.Tag}, \text{FSMAC.Ver})$ a key-evolving MAC scheme, and $\text{CAEAD} = (\text{CAEAD.Gen}, \text{CAEAD.Enc}, \text{CAEAD.Dec}, \text{CAEAD.Ver})$ a CAEAD scheme. We assume that outputs of sPRG can be used as secret keys of CAEAD. From these, we construct our key-evolving message franking scheme $\text{FSMF} = (\text{SKGen}, \text{TKGen}, \text{SKUpd}, \text{TKUpd}, \text{Enc}, \text{Tag}, \text{Dec}, \text{Ver})$ in Figure 1.

The correctness of the scheme immediately follows from the correctness of CAEAD and FSMAC.

4.2 Security Proof

In this section, we show that our construction of FSMF given in Section 4.1 satisfies security notions defined in Section 3.2.

Theorem 1 (FS-MO-IND Security). *If sPRG satisfies forward security and CAEAD satisfies MO-IND security, then FSMF satisfies FS-MO-IND security.*

Proof. Let \mathcal{A} be a PPT adversary that attacks the FS-MO-IND security of FSMF. We introduce the following games $Game_\alpha$ for $\alpha = 0, 1$.

- $Game_0$: $Game_0$ is exactly the same as the game of FS-MO-IND security.
- $Game_1$: $Game_1$ is identical to $Game_0$ except that \mathcal{CH} computes $Out_i \xleftarrow{\$} \{0, 1\}^\lambda$ after computing $(Out'_i, St_i) \leftarrow \text{sPRG.Next}(St_{i-1})$.

Let G_α be the event that \mathcal{A} succeeds in guessing the challenge bit in $Game_\alpha$.

Lemma 1. *There exists a PPT adversary \mathcal{B} such that $|\Pr[G_0] - \Pr[G_1]| = 2 \cdot \text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda)$.*

Proof. We construct an adversary \mathcal{B} that attacks the forward security of sPRG, using the adversary \mathcal{A} as follows.

1. \mathcal{B} sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. Upon receiving Out_i from \mathcal{CH} , \mathcal{B} sets $i := i + 1$.
3. \mathcal{B} answers encryption queries and decryption queries from \mathcal{A} as follows.
 - On encryption queries of the form (H, M) , \mathcal{B} computes $(C_1, C_2) \leftarrow \text{CAEAD.Enc}(Out_i, H, M)$, returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
 - On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $(H, (C_1, C_2, \hat{i})) \notin S_i$, \mathcal{B} returns \perp to \mathcal{A} . Otherwise, \mathcal{B} computes $(M', O) \leftarrow \text{CAEAD.Dec}(Out_i, H, C_1, C_2)$ and returns (M', O) to \mathcal{A} .
4. When \mathcal{A} outputs $(d, H^*, M_0^*, M_1^*, i^*)$, \mathcal{B} returns d to \mathcal{CH} .
5. \mathcal{B} receives St_i from \mathcal{CH} , sets $\text{SK}_i = (i, Out_i, St_i)$, and chooses $g \xleftarrow{\$} \{0, 1\}$.
6. If $i^* \geq i$, \mathcal{B} sets $b' := g$, returns b' to \mathcal{CH} , and terminates. Otherwise, \mathcal{B} computes $(C_1^*, C_2^*) \leftarrow \text{CAEAD.Enc}(Out_{i^*}, H^*, M_g^*)$ and returns $(\text{SK}_i, (C_1^*, C_2^*, i^*))$ to \mathcal{A} .
7. When \mathcal{A} outputs g' , if $g' = g$, \mathcal{B} sets $b' := 1$, else $b' := 0$.
8. \mathcal{B} returns b' to \mathcal{CH} .

We can see that \mathcal{B} perfectly simulates the game $Game_0$ if $b = 1$ and $Game_1$ if $b = 0$ for \mathcal{A} . We assume that G_α occurs. Then, \mathcal{B} outputs $b' = 1$ since \mathcal{A} succeeds in guessing the challenge bit g in $Game_\alpha$. Thus, $\text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda) = \frac{1}{2} \cdot |\Pr[b' = 1|b = 1] - \Pr[b' = 1|b = 0]| = \frac{1}{2} \cdot |\Pr[G_0] - \Pr[G_1]|$ holds. \square

Lemma 2. *There exists a PPT adversary \mathcal{D} such that $|\Pr[G_1] - \frac{1}{2}| = n \cdot \text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-IND}}(\lambda)$.*

Proof. We construct an adversary \mathcal{D} that attacks the MO-IND security of CAEAD, using the adversary \mathcal{A} as follows.

1. \mathcal{D} computes $l \leftarrow [n]$ and $St_0 \leftarrow \text{sPRG.Key}(1^\lambda, n)$ and sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{D} sets $i := i+1$, computes $(Out'_i, St_i) \leftarrow \text{sPRG.Next}(St_{i-1})$ and $Out_i \xleftarrow{\$} \{0, 1\}^\lambda$ and sets $SK_i := (i, Out_i, St_i)$.
3. \mathcal{D} answers encryption queries and decryption queries from \mathcal{A} as follows.
 - If $i = l$, on encryption queries of the form (H, M) , \mathcal{D} makes encryption queries of the form (H, M) to \mathcal{CH} , gets the result (C_1, C_2) , returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $(H, (C_1, C_2, \hat{i})) \notin S_i$, \mathcal{D} returns \perp to \mathcal{A} . Otherwise, \mathcal{D} makes decryption queries of the form (H, C_1, C_2) to \mathcal{CH} , gets the result (M', O) , and returns (M', O) to \mathcal{A} .
 - If $i \neq l$, on encryption queries of the form (H, M) , \mathcal{D} computes $(C_1, C_2) \leftarrow \text{CAEAD.Enc}(Out_i, H, M)$, returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $(H, (C_1, C_2, \hat{i})) \notin S_i$, \mathcal{D} returns \perp to \mathcal{A} . Otherwise, \mathcal{D} computes $(M', O) \leftarrow \text{CAEAD.Dec}(Out_i, H, C_1, C_2)$ and returns (M', O) to \mathcal{A} .
4. \mathcal{A} outputs $(d, H^*, M_0^*, M_1^*, i^*)$.
5. If $d = 1$ or $i = n$, \mathcal{D} proceeds to the next Step, else repeats Steps 2 through 4.
6. If $i^* \neq l$, \mathcal{D} chooses $b' \xleftarrow{\$} \{0, 1\}$, returns b' to \mathcal{CH} , and terminates. Otherwise, \mathcal{D} returns (H^*, M_0^*, M_1^*) to \mathcal{CH} .
7. \mathcal{D} receives (C_1^*, C_2^*) from \mathcal{CH} and returns $(SK_i, (C_1^*, C_2^*, i^*))$ to \mathcal{A} .
8. When \mathcal{A} outputs g' , \mathcal{D} sets $b' := g'$ and returns b' to \mathcal{CH} .

We can see that \mathcal{D} perfectly simulates the game $Game_1$ for \mathcal{A} . We assume that G_1 occurs and $i^* = l$ holds or $i^* \neq l$ holds and the challenge bit matches the bit chosen randomly. Then, \mathcal{D} succeeds in guessing the challenge bit in MO-IND security game. Since probability of $i^* = l$ is $\frac{1}{n}$ and probability that the challenge bit matches the bit chosen randomly is $\frac{1}{2}$, $\text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-IND}}(\lambda) = \left| \left(\frac{1}{n} \Pr[G_1] + \frac{n-1}{n} \cdot \frac{1}{2} \right) - \frac{1}{2} \right| = \frac{1}{n} \cdot \left| \Pr[G_1] - \frac{1}{2} \right|$ holds. \square

Combining Lemma 1 and 2, We have

$$\begin{aligned} \text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-IND}}(\lambda) &= \left| \Pr[G_0] - \frac{1}{2} \right| \\ &\leq |\Pr[G_0] - \Pr[G_1]| + \left| \Pr[G_1] - \frac{1}{2} \right| \\ &= 2 \cdot \text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda) + n \cdot \text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-IND}}(\lambda), \end{aligned}$$

which concludes the proof of Theorem 1. \square

Theorem 2 (FS-MO-CTXT Security). *If sPRG satisfies forward security and CAEAD satisfies MO-CTXT security, then FSMF satisfies FS-MO-CTXT security.*

Proof. Let \mathcal{A} be a PPT adversary that attacks the FS-MO-CTXT security of FSMF. We introduce the following games $Game_\alpha$ for $\alpha = 0, 1$.

- $Game_0$: $Game_0$ is exactly the same as the game of FS-MO-CTXT security.
- $Game_1$: $Game_1$ is identical to $Game_0$ except that \mathcal{CH} computes $Out_i \xleftarrow{\$} \{0, 1\}^\lambda$ after computing $(Out'_i, St_i) \leftarrow \text{sPRG.Next}(St_{i-1})$.

Let G_α be the event that \mathcal{A} succeeds in outputting the tuple $(H^*, (C_1^*, C_2^*, i^*))$ satisfying

$$M^* \neq \perp, (H^*, (C_1^*, C_2^*, i^*)) \notin S_{i^*}, \text{ and } 1 \leq i^* < i$$

in computing $(M^*, O^*) \leftarrow \text{Dec}(\text{SK}_{i^*}, H^*, (C_1^*, C_2^*, i^*))$ in $Game_\alpha$.

Lemma 3. *There exists a PPT adversary \mathcal{B} such that $|\Pr[G_0] - \Pr[G_1]| = 2 \cdot \text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda)$.*

Proof. We construct an adversary \mathcal{B} that attacks the forward security of sPRG, using the adversary \mathcal{A} as follows.

1. \mathcal{B} sets $i := 0$ and $S_t := \emptyset$ for $t = [n]$.
2. Upon receiving Out_i from \mathcal{CH} , \mathcal{B} sets $i := i + 1$.
3. \mathcal{B} answers encryption queries and decryption queries from \mathcal{A} as follows.
 - On encryption queries of the form (H, M) , \mathcal{B} computes $(C_1, C_2) \leftarrow \text{CAEAD.Enc}(Out_i, H, M)$, returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
 - On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $i \neq \hat{i}$, \mathcal{B} returns \perp to \mathcal{A} . Otherwise, \mathcal{B} computes $(M', O) \leftarrow \text{CAEAD.Dec}(Out_i, H, C_1, C_2)$ and returns (M', O) to \mathcal{A} .
4. When \mathcal{A} outputs d , \mathcal{B} returns d to \mathcal{CH} .
5. \mathcal{B} receives St_i from \mathcal{CH} , sets $\text{SK}_i := (i, Out_i, St_i)$, and returns SK_i to \mathcal{A} .
6. When \mathcal{A} outputs $(H^*, (C_1^*, C_2^*, i^*))$, \mathcal{B} computes $(M^*, O^*) \leftarrow \text{CAEAD.Dec}(Out_{i^*}, H^*, C_1^*, C_2^*)$. If $M^* \neq \perp$, $(H^*, (C_1^*, C_2^*, i^*)) \notin S_{i^*}$, and $1 \leq i^* < i$, \mathcal{B} sets $b' := 1$, else $b' := 0$.
7. \mathcal{B} returns b' to \mathcal{CH} .

We can see that \mathcal{B} perfectly simulates the game $Game_0$ if $b = 1$ and $Game_1$ if $b = 0$ for \mathcal{A} . We assume that G_α occurs. Then, \mathcal{B} outputs $b' = 1$ since \mathcal{A} succeeds in outputting the tuple $(H^*, (C_1^*, C_2^*, i^*))$ satisfying

$$M^* \neq \perp, (H^*, (C_1^*, C_2^*, i^*)) \notin S_{i^*}, \text{ and } 1 \leq i^* < i$$

in computing $(M^*, O^*) \leftarrow \text{Dec}(\text{SK}_{i^*}, H^*, (C_1^*, C_2^*, i^*))$ in $Game_\alpha$. Thus, $\text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda) = \frac{1}{2} \cdot |\Pr[b' = 1 | b = 1] - \Pr[b' = 1 | b = 0]| = \frac{1}{2} \cdot |\Pr[G_0] - \Pr[G_1]|$ holds. \square

Lemma 4. *There exists a PPT adversary \mathcal{D} such that $\Pr[G_1] = n \cdot \text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-CTXT}}(\lambda)$.*

Proof. We construct an adversary \mathcal{D} that attacks the MO-CTXT security of CAEAD, using the adversary \mathcal{A} as follows.

1. \mathcal{D} computes $l \leftarrow [n]$ and $St_0 \leftarrow \text{sPRG.Key}(1^\lambda, n)$ and sets $i := 0$ and $S_t := \emptyset$ for all $t \in [n]$.
2. \mathcal{D} sets $i := i + 1$, computes $(Out'_i, St_i) \leftarrow \text{sPRG.Next}(St_{i-1})$ and $Out_i \leftarrow \{0, 1\}^\lambda$, and sets $SK_i := (i, Out_i, St_i)$.
3. \mathcal{D} answers encryption queries and decryption queries from \mathcal{A} as follows.
 - If $i = l$, on encryption queries of the form (H, M) , \mathcal{D} makes encryption queries of the form (H, M) to \mathcal{CH} , gets the result (C_1, C_2) , returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $i \neq \hat{i}$, \mathcal{D} returns \perp to \mathcal{A} . Otherwise, \mathcal{D} makes decryption queries of the form (H, C_1, C_2) to \mathcal{CH} , gets the result (M', O) , and returns (M', O) to \mathcal{A} .
 - If $i \neq l$, on encryption queries of the form (H, M) , \mathcal{D} computes $(C_1, C_2) \leftarrow \text{CAEAD.Enc}(Out_i, H, M)$, returns (C_1, C_2, i) to \mathcal{A} , and appends $(H, (C_1, C_2, i))$ to S_i .
On decryption queries of the form $(H, (C_1, C_2, \hat{i}))$, if $i \neq \hat{i}$, \mathcal{D} returns \perp to \mathcal{A} . Otherwise, \mathcal{D} computes $(M', O) \leftarrow \text{CAEAD.Dec}(Out_i, H, C_1, C_2)$ and returns (M', O) to \mathcal{A} .
4. \mathcal{A} outputs d .
5. If $d = 1$ or $i = n$, \mathcal{D} proceeds to the next Step, else repeats Steps 2 through 4.
6. If $i \leq l$, \mathcal{D} terminates, else returns SK_i to \mathcal{A} .
7. When \mathcal{A} outputs $(H^*, (C_1^*, C_2^*, i^*))$, if $i^* = l$, \mathcal{D} returns (H^*, C_1^*, C_2^*) to \mathcal{CH} , else terminates.

We can see that \mathcal{D} perfectly simulates the game $Game_1$ for \mathcal{A} . We assume that G_1 occurs and $i^* = l$ holds. Then, \mathcal{D} successfully outputs the tuple (H^*, C_1^*, C_2^*) satisfying

$$M^* \neq \perp \quad \text{and} \quad (H^*, C_1^*, C_2^*) \notin S$$

in computing $(M^*, O) \leftarrow \text{CAEAD.Dec}(K, H^*, C_1^*, C_2^*)$ in MO-CTXT security game. Since probability of $i^* = l$ is $\frac{1}{n}$, $\text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-CTXT}}(\lambda) = \frac{1}{n} \cdot \Pr[G_1]$ holds. \square

Combining Lemma 3 and 4, We have

$$\begin{aligned} \text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-MO-CTXT}}(\lambda) &= \Pr[G_0] \\ &\leq |\Pr[G_0] - \Pr[G_1]| + \Pr[G_1] \\ &= 2 \cdot \text{Adv}_{\text{sPRG}, \mathcal{B}}^{\text{FS-PRG}}(\lambda) + n \cdot \text{Adv}_{\text{CAEAD}, \mathcal{D}}^{\text{MO-CTXT}}(\lambda), \end{aligned}$$

which concludes the proof of Theorem 2. \square

Theorem 3 (FS-UNF Security). *If FSMAC satisfies FS-sEUF-CMA security, then FSMF satisfies FS-UNF security.*

Proof. Let \mathcal{A} be a PPT adversary that attacks the FS-UNF security of FSMF. We construct an adversary \mathcal{B} that attacks the FS-sEUF-CMA security of FSMAC, using the adversary \mathcal{A} as follows.

1. \mathcal{B} sets $j := 0$ and $T_t := \emptyset$ for all $t \in [n]$.

2. \mathcal{B} sets $j := j + 1$.
3. \mathcal{B} answers tagging queries of the form C_2 from \mathcal{A} as follows. \mathcal{B} makes tagging queries of the form C_2 to \mathcal{CH} , gets the result (τ, j) , returns (τ, j) to \mathcal{A} , and appends $(C_2, (\tau, j))$ to T_j .
4. When \mathcal{A} outputs d , \mathcal{B} returns d to \mathcal{CH} .
5. \mathcal{B} receives K_j from \mathcal{CH} , sets $\text{TK}_j = K_j$, and returns TK_j to \mathcal{A} .
6. When \mathcal{A} outputs $(H^*, M^*, O^*, C_2^*, (\tau^*, j^*))$, \mathcal{B} returns $(C_2^*, (\tau^*, j^*))$ to \mathcal{CH} , else terminates.

We can see that \mathcal{B} perfectly simulates the FS-sEUF-CMA security game for \mathcal{A} . We assume that \mathcal{A} successfully outputs the tuple $(H^*, M^*, O^*, C_2^*, (\tau^*, j^*))$ satisfying

$$\text{Ver}(\text{TK}_{j^*}, H^*, M^*, O^*, C_2^*, (\tau^*, j^*)) = 1, (C_2^*, (\tau^*, j^*)) \notin T_{j^*}, \text{ and } 1 \leq j^* < j.$$

Then, \mathcal{B} successfully outputs $(C_2^*, (\tau^*, j^*))$ satisfying

$$\text{FSMAC.Ver}(K_{j^*}, C_2^*, (\tau^*, j^*)) = 1 \text{ and } (C_2^*, (\tau^*, j^*)) \notin S_{j^*}$$

in FS-sEUF-CMA security game. Thus, $\text{Adv}_{\text{FSMAC}, \mathcal{B}}^{\text{FS-sEUF-CMA}}(\lambda) = \text{Adv}_{\text{FSMF}, \mathcal{A}}^{\text{FS-UNF}}(\lambda)$, which concludes the proof of Theorem 3. \square

Theorem 4 (FS-R-BIND Security). *If CAEAD satisfies R-BIND, then FSMF satisfies FS-R-BIND security.*

Proof. Let \mathcal{A} be a PPT adversary that attacks the FS-R-BIND of FSMF. We assume that \mathcal{A} successfully outputs the tuple $((H, M, O), (H', M', O'), C_2^*, (\tau^*, j^*))$ satisfying

$$\begin{aligned} \text{Ver}(\text{TK}_{j^*}, H, M, O, C_2^*, (\tau^*, j^*)) &= \text{Ver}(\text{TK}_{j^*}, H', M', O', C_2^*, (\tau^*, j^*)) = 1, \\ (H, M) &\neq (H', M'), \text{ and } 1 \leq j^* < j. \end{aligned}$$

Then, $\text{CAEAD.Ver}(H, M, O, C_2^*) = \text{CAEAD.Ver}(H', M', O', C_2^*) = 1$ and $(H, M) \neq (H', M')$ holds. This contradicts the R-BIND security of CAEAD. Thus, Theorem 4 holds. \square

Theorem 5 (FS-S-BIND security). *If CAEAD satisfies S-BIND, then FSMF satisfies FS-S-BIND security.*

Proof. Let \mathcal{A} be a PPT adversary that attacks the FS-S-BIND of FSMF. We assume that \mathcal{A} successfully outputs the tuple $(j^*, \text{SK}_i, H, C_1, C_2)$ satisfying

$$\text{Ver}(\text{TK}_{j^*}, H, M', O, C_2, (\tau, j^*)) = 0, M' \neq \perp, \text{ and } 1 \leq j^* < j$$

in computing $(\tau, j^*) \leftarrow \text{Tag}(\text{TK}_{j^*}, C_2)$ and $(M', O) \leftarrow \text{Dec}(\text{SK}_i, H, C_1, C_2)$.

When $\text{Ver}(\text{TK}_{j^*}, H, M', O, C_2, (\tau, j^*)) = 0$ holds, at least one of $\text{CAEAD.Ver}(H, M', O, C_2) = 0$ and $\text{FSMAC.Ver}(K_{j^*}, C_2, \tau) = 0$ holds.

If $\text{FSMAC.Ver}(K_{j^*}, C_2, \tau) = 0$ holds, $\text{FSMAC.Ver}(K_{j^*}, C_2, \text{FSMAC.Tag}(K_{j^*}, C_2)) = 0$ holds. This contradicts the correctness of FSMAC.

If $\text{CAEAD.Ver}(H, M', O, C_2) = 0$ holds, $M' \neq \perp$ and $\text{CAEAD.Ver}(H, M, O, C_2) = 0$ holds in computing $(M', O) \leftarrow \text{CAEAD.Dec}(Out_i, H, C_1, C_2)$. This contradicts the S-BIND security of CAEAD. Thus, Theorem 5 holds. \square

5 Conclusion

In this work, we propose forward secure message franking. Firstly, we formalize key-evolving message franking including additional key update algorithms. Then, we propose forward security for five security requirements. Finally, we show key-evolving message franking satisfying forward security based on committing authenticated encryption with associated data, forward secure pseudorandom generator, and forward secure message authentication code.

Our definition is based on CAEAD introduced by Grubbs et al. [13]. In [6, 18], variants of CAEAD were also proposed. By applying forward secure PRG and forward secure MAC to these schemes as a similar manner to our construction, it seems that forward secure variants schemes are obtained.

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