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Comparison of theoretical approach and Monte Carlo approach for uncertainty analysis in probabilistic safety assessment

Gyun Seob Song, Man Cheol Kim*
School of Energy Systems Engineering, Chung-Ang University, 84 Heukseok-ro, Dongjak-gu, Seoul, 06974, Korea
**Corresponding author: charleskim@cau.ac.kr*

1. Introduction

Probabilistic safety assessment (PSA) is a method to estimate quantitative risks and make decision associated with complex systems. One of major elements in the PSA is quantifying uncertainties called uncertainty analysis. In the uncertainty analysis, major sources of uncertainties are state-of-knowledge uncertainties which are interpreted as our degree of beliefs (probability) regarding numerical values of model parameters like failure probability [1]. State-of-knowledge uncertainties are expressed with subjective probability density functions in these days. Uncertainties in the system characteristics are determined by synthesizing these probability density functions associated with component characteristics. There are many methods that have been developed for synthesizing probability density functions [2]. Among the methods, Monte Carlo simulation is generally used in PSA because it can estimate numerical value with high accuracy when probability density functions are well-defined. In rare events, however, sample size should be increased to ensure accuracy. In the literature, joint statistical method is an effective method to solve above problem [3]. In this paper, the joint statistical method is applied to an example system and the results are compared with those with Monte Carlo approach.

2. Theory and Method

Song and Kim [4] provides details of how the probability density functions for AND and OR logics of two basic events can be derived. Analytic solutions on the probability density functions of AND and OR logics, $f_{AND}(t)$ and $f_{OR}(t)$, are given as follows:

$$f_{AND}(t) = \int_t^1 \frac{1}{x} f_X(x) f_Y(t/x) dx \qquad (1)$$

$$f_{OR}(t) = \int_0^t \frac{1}{1-x} f_X(x) f_Y\left(\frac{t-x}{1-x}\right) dx \qquad (2)$$

It is possible to obtain any system probability density function by repeatedly applying Eq. (1) and Eq. (2). Numerical integration is used to find the probability density function of the top event. Absolute tolerance is set to 0 and relative tolerance is set to 0.1%, because relative tolerance determines the accuracy of the integral in the case of small value. The numerical integration is performed with a newly developed MATLAB™ code.

3. Application to an example system

3.1 Example system and fault tree model

Fig. 1 shows an example auxiliary feedwater system (AFWS) which is a system to provide adequate feedwater from condensate storage tank (CST) to a steam generator (SG) to allow continued residual heat removal from the primary system when main feedwater system is not available. AFWS include two pumps to provide adequate feedwater, a motor driven pump (MDP) and a turbine driven pump (TDP). Even when one of the pumps is unavailable, the system is still available because each pump has the capacity to provide enough feedwater to a SG. This is equal to logical conjunction, which means that system is unavailable only when all of the components are unavailable. Unavailability from hardware failure of a component is generally due to standby failure and running failure [5]. This is equal to logical disjunction because component is unavailable when any failure mode event occurs. Fig. 2 shows the fault tree for the example AFWS with two pumps.

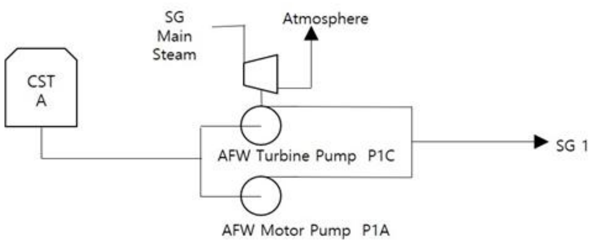

Fig. 1. Example AFWS for a SG

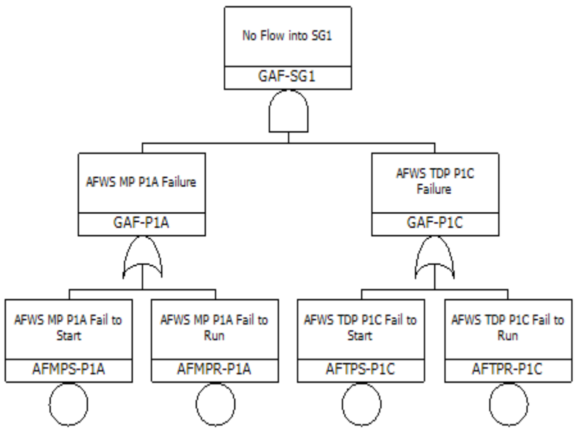

Fig. 2. Fault tree for the example AFWS

3.2 Reliability data

State-of-knowledge uncertainties has been improved by subjective (Bayesian) probability theory and the experience from industries. PSAs of nuclear power plants have used various statistical distributions to express uncertainties. Lognormal distribution has been generally used in many studies before [6]. But more recent studies performed at Idaho National Laboratory used beta and gamma distributions because the two distributions match the bounds for probabilities and occurrence rates, respectively [7]. AIMS-PSA provides above three types of distributions for uncertainty analysis. To compare various distribution cases, the joint statistic method and Monte Carlo simulation are applied to different reliability data. Table I and Table II provides industry-average data of a MDP and a TDP and their failure modes. In fail-to-run cases, mission time is assumed as 24 hours.

Table I: Reliability data from EGG-SSRE-8875 [6]

Failure Mode	Type	Parameters	
		μ_x	EF
Motor-Driven Pump Fail to Start	Lognormal	3E-3	5
Motor-Driven Pump Fail to Run	Lognormal	7.2E-4	10
Turbine-Driven Pump Fail to Start	Lognormal	3E-2	5
Turbine Driven Pump Fail to Run	Lognormal	2.4E-3	10

Table II: Reliability data from NUREG/CR-6928 [7]

Failure Mode	Type	Parameters	
		α	β
Motor-Driven Pump Fail to Start	Beta	0.909	617.5
Motor-Driven Pump Fail to Run	Gamma	0.5	3591.3
Turbine-Driven Pump Fail to Start	Beta	0.414	59.76
Turbine Driven Pump Fail to Run	Gamma	0.5	283.46

4. Result and Discussion

State-of-knowledge uncertainties are generally expressed in subjective probabilities because identical trial is impossible in real world. However, as a mind construct or a simulation tool, the occurrence probability is interpreted as the degree of belief regarding numerical values of model parameters [8]. Fig. 3 and Fig. 4 show the occurrence probabilities for joint statistic method and Monte Carlo simulation associated with the unavailability of the example AFWS with reliability data of Table I and Table II, respectively. Monte Carlo simulation results are provided by AIMS-PSA, one of integration and quantification tool of PSA, with 100,000 sample size in each simulation. In joint statistic method, occurrence probability is calculated by the trapezoidal rule.

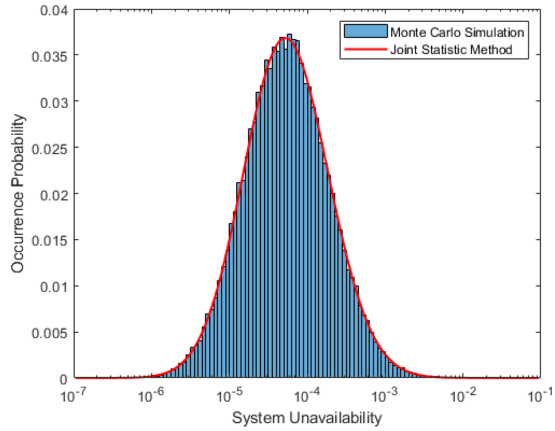

Fig. 3. Occurrence probability for unavailability of example AFWS with reliability data from EGG-SSRE-8875

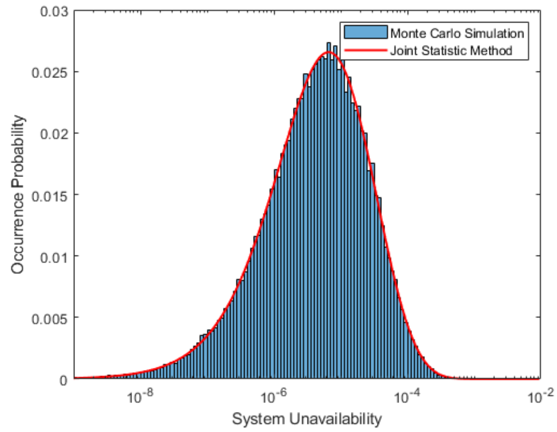

Fig. 4. Occurrence probability for unavailability of example AFWS with reliability data from NUREG/CR-6928

5. Conclusion

In this paper, joint statistic method for synthesizing probability density functions to quantify system uncertainty is presented. Joint statistic method is relatively more accurate compared with Monte Carlo simulation. But there is still internal uncertainty in numerical integration because of tolerance error. Also, the running time to calculate numerical integration increases as the system becomes complex and has more random variables. If there is more effective method to calculate numerical integration, the accuracy of the result and running time can be improved.

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