Integration of Fluid Dynamics and Solid Mechanics Models for FSI Simulation using GPU-based SPH Framework

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1. Introduction

In many engineering fields, pressure from fluid flow causes major deformation in the structure. This interaction is Fluid Structure Interaction (FSI). Among the many ways to describe the FSI are the Arbitrary Lagrangian Eulerian (ALE) typical. However, ALE has disadvantages in phenomena such as large deformation of the structure or rapid flow of fluid [1]. On the other hand, the Smoothed Particle Hydrodynamics (SPH) method, which is a Fully Lagrangian method, has advantage to this interpretation.

FSI is a method used in many areas of nuclear engineering, such as the behavior of the fuel assembly flowing axially in the direction of coolant, pressure loads on the vessel internal structures in PWR during a LOCA, blowdown, Flow-Induced Vibration (FIV), flow-induced fluid-elastic vibrations, sloshing of pressurizer on a nuclear ship, rupture or swelling of fuel rods and In-Vessel Retention (IVR) failed due to broken vessel. This is because accurately simulating the interaction of fluid and structure can help design the plant and cope with accidents.

In this study, the interaction of fluid with structure was added to the SOPHIA code to implement the FSI. The SOPHIA code is an SPH-based parallelization multi-physics code developed by Seoul National University [2]. To validation this, Benchmark experiment and numerical simulation were compared with simulation through SOPHIA code with FSI added.

2. SPH Method

2.1 Concept of SPH method

Smoothed Particle Hydrodynamics was first developed in astrophysics as a meshless CFD method of the Lagrangian-based [3]. The fluid is represented as a collection of finite particles in a way that tracks and interprets fluid motion, not based on space and grid. Particles that are considered to be a collection of fluid molecules move along with physical quantities (mass, velocity, temperature, etc.) that are determined by the type of fluid and the spacing of particles. Track the movement of particles by setting up initial conditions for them and interpreting interactions with the central particle and its surrounding particles. The SPH method has the advantage of dealing with undetermined areas of interpretation or highly variable flows, thanks to the nature of the Lagrangian-based analysis method.

2.2 SPH Approximation

Mathematically, random functions can be expressed in integral form using delta functions. The SPH method is represented in integral form using the kernel function, a continuous function with a smoothing length h instead of a delta function. Discretizing this integral form is like (1).

$$f_i(r) = \sum_j f_j W(r_{ij}, h) V_j \tag{1}$$

Where $\boldsymbol{i}$ stands for the central particle and $\boldsymbol{j}$ stands for the surrounding particle. $\boldsymbol{W(r_{ij}, h)}$ represents the kernel function, $\boldsymbol{r_{ij} = r_i - r_j}$, $\boldsymbol{V_j}$ is volume of adjacent particle and $\boldsymbol{h}$ is the smoothing length indicating the range of nearby particles to be included in the approximation process. The kernel function must be able to approximate the delta function, so it has a very large value at the center and the farther away from the center, the more convergent it is to zero. It also satisfies all the mathematical properties of the delta function.

There are several ways to obtain a gradient of function, depending on the method of deriving. In this study, a gradient of function was calculated in the following manner

$$\nabla f_i(r) = \rho_i \sum_j m_j \left(\frac{f_i + f_j}{\rho_i \rho_j} \right) \nabla W(r_{ij}, h) \tag{2}$$

3. Fluid Structure Interaction (FSI)

3.1 Fluid Dynamics

In the SPH method, there are two methods for solving mass conservation equations: mass summation method and solving continuity equation method. The mass summation method was used in this study.

$$\rho_i(r) = \sum_j m_j W(r_{ij}, h) \tag{2}$$

The momentum conservation equation is described in Lagrangian form as (3).

$$\rho \frac{\overrightarrow{du}}{dt} = -\nabla p + \mu \nabla^2 \vec{u} + \rho \vec{g} \tag{3}$$

Where $\boldsymbol{\rho}$ is density, $\vec{\boldsymbol{u}}$ is speed, $\boldsymbol{p}$ is pressure, $\vec{\boldsymbol{g}}$ is gravity acceleration and $\boldsymbol{\mu}$ is dynamic viscosity. The first term on the right hand side represents the pressure force and the second term is the viscous force, which is (4) and (5) if they are to be discretized.

$$\left(\frac{\overrightarrow{du}}{dt}\right)_{fp,i} = -\sum_j m_j \left(\frac{p_i+p_j}{\rho_i\rho_j}\right)\nabla W_{ij} \quad (4)$$

$$\left(\frac{\overrightarrow{du}}{dt}\right)_{fv,i} = \sum_j \frac{4m_j\mu_j\overrightarrow{r_{ij}}\cdot\nabla W_{ij}}{(\rho_i+\rho_j)\left(|\overrightarrow{r_{ij}}|^2+\eta^2\right)}\left(\overrightarrow{u_i}-\overrightarrow{u_j}\right) \quad (5)$$

The pressure is calculated as Equation of State (EOS) using the Tait's equation [4]. This equation is given as shown in (6) in a manner that assumes weakly compressible.

$$\mathrm{p} = \frac{c_0^2\rho_0}{\gamma}\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] \quad (6)$$

Where $\boldsymbol{\rho_0}$ is the reference density of the fluid, $\boldsymbol{c_0}$ is speed of sound and $\gamma = 7$.

3.2 Structure Dynamics

Elastic force is calculated using the divergence of the stress tensor. In order to calculate these stress tensors, we must first calculate the F (Deformation gradient). The change of the solid consists of three types: translation, deformation, and rotation. Elastic force depends solely on deformation, so translation and rotation should be extracted from the change of the solid. F (Deformation gradient) indicates how much of the current position has been changed from the initial position. At this time, F excludes only the effects of translation from the three changes in the solid. This is as shown in equation (7) when SPH approximation is performed. In contrast to A. Peer et al [5], F was calculated without using a kernel gradient correction.

$$\mathbf{F}_\mathrm{i} = \sum_j^0 V_j^0 \mathbf{X}_{ji} \otimes \nabla W_i\left(\mathbf{X}_{ij}^0\right). \quad (7)$$

Where $\mathbf{X}_{ji} = \mathbf{X}_j - \mathbf{X}_i$, $\mathbf{X}_i$ is position vector and the superscript 0 is initial value.

R (Rotation matrix) must be calculated first to exclude rotation from F. Instead of calculating R directly for each step, calculate R through iteration using R of the previous step. Iteration is performed until ω is less than 1e-10, where ω is the vector where $\|\mathrm{F} - R\|^2$ is the minimum. R of the next step is calculated by rotating R of the previous step in the same direction as ω in the angle of ω [6].

$$\mathbf{R} \leftarrow \exp(\boldsymbol{\omega})\mathbf{R}, \mathbf{R} \leftarrow \exp\left(\frac{\sum_i r_i \times a_i}{|\sum_i r_i \cdot a_i| + \varepsilon}\right)\mathbf{R}. \quad (8)$$

Calculate the $\mathbf{F}^*$ (Corotated deformation gradient), which removes the change by rotation from F, to take into account only the effects of the deformation. $\mathbf{F}^*$ is calculated as the difference between the change of the current position and the change of the initial position multiplied by the rotation. Finally, only the effect of the deformation will remain in the structure [5].

$$\mathbf{F}_i^* = \mathbf{I} + \sum_j^0 V_j^0 \left(\mathbf{X}_{ji} - \boldsymbol{R}_i\mathbf{X}_{ji}^0\right) \otimes \nabla W_i^*\left(\mathbf{X}_{ij}^0\right). \quad (9)$$

$$\nabla W_i^*\left(\mathbf{X}_{ij}^0\right) = \boldsymbol{R}_i \nabla W_i\left(\mathbf{X}_{ij}^0\right).$$

Calculate the strain according to the linear elasticity model from the $\mathbf{F}^*$.

$$\varepsilon_i = \frac{1}{2}\left(\mathbf{F}_i^* + \mathbf{F}_i^{*T}\right) - \mathbf{I}. \quad (10)$$

Calculate the stress through the Piola-Kirchhoff stress tensor using strain. G and K stand for Shear and Bulk modulus, respectively.

$$\boldsymbol{P}_i = 2G\varepsilon_i + \left(K - \frac{2}{3}G\right)tr(\varepsilon_i)\mathbf{I}. \quad (11)$$

Finally, calculate the elastic force through the divergence of the stress sensor [5].

$$\mathbf{f}_i = \sum_j^0 V_i^0 V_j^0 \left(\mathbf{P}_i \nabla W_i^*\left(\mathbf{X}_{ij}^0\right) - \mathbf{P}_j \nabla W_j^*\left(\mathbf{X}_{ji}^0\right)\right). \quad (12)$$

$\nabla W_j = -\nabla W_i$ allows the expression to be described as (4), but in case of (12), the kernel function multiplied by the R of each particle, the expression is described separately as (12).

3.3 Fluid Structure Coupling

Fluid and structure are calculated only between the same kinds of particles, so additional forces acting on each other should be calculated. In this study pressure force was used as the interaction force between Fluid and Structure.

Based on the points calculated using pressure force in the relationship between the Fluid particle and the Boundary particle, simulation code was configured to push each other with pressure force even in the relationship between the fluid and structure.

First, use EOS to calculate the pressure from the structure particle, as with the boundary particle. Then, for one central structure particle, the pressure force (f_P) is calculated if the adjacent particle is fluid particle and the elastic force (f_e) if the adjacent particle is structure particle. It is described in Fig.1.

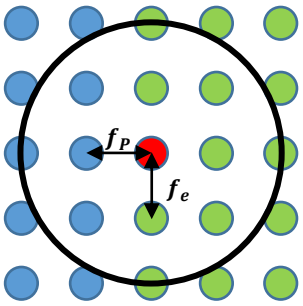

Fig. 1 Fluid Structure Interaction Force

4. SPH Simulation on Deformation of Elastic Plate

4.1. Experimental set up

This study simulated the deformation of elastic plate by the pressure of fluid. The Dam Breaking Through a Rubber gate experiment performed by Antoci et al. [7] was selected as the benchmark case and validation was performed compared to the numerical simulation of Zhan et al. [8]. This experiment is an experiment in which the water of the tank flows out of the elastic gate and the initial shape and conditions are as shown in Fig.2.

The total number of particles used in the simulation is 1,024,256, with 700,000 fluid particles, 20,250 structural particles and 304,006 boundary particles. From a numerical perspective, the number of particles (particle spacing) in SPH analysis refers to the resolution of interpolation. SPH analysis is affected by resolution so that it is possible to handle as many particles as possible within the physically possible time range.

Bulk modulus K is 1.9e7, shear modulus G is 1.96e6, damping coefficient α is 0.04 and the density for fluid and structure are 1000kg/ m^3 and 1,100kg/ m^3 , respectively [8]. Because it is an analysis for the purpose of code validation, the analysis was performed in exactly the same properties as the benchmark case. The simulation features are shown in Fig. 3.

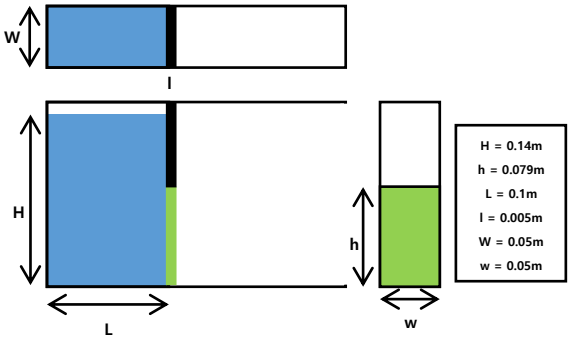

Fig. 2 Simulation Initial Condition

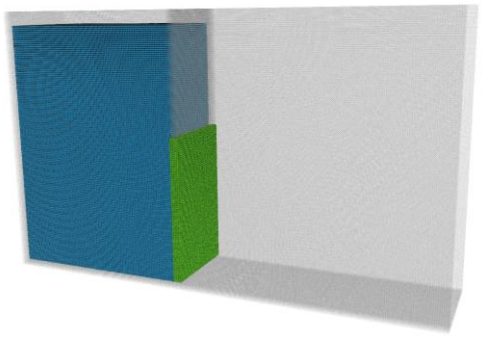

Fig. 3 Simulation Initial Geometry

4.2. Simulation result

When the simulation starts, the fluid pushes the elastic gate under pressure and the fluid flows through the opening gap. Initially, the horizontal and vertical displacement of the gate rises rapidly from 0.0s to 0.16s, and then gradually decreases. Simulations were performed up to 0.4s because the experiment was carried out up to 0.4s. Fig. 4 is a qualitatively comparison of experimental scenes over time and simulation scenes in the same time zone. A total of 6 times were compared from 0.0s to 0.4s intervals of 0.08s.

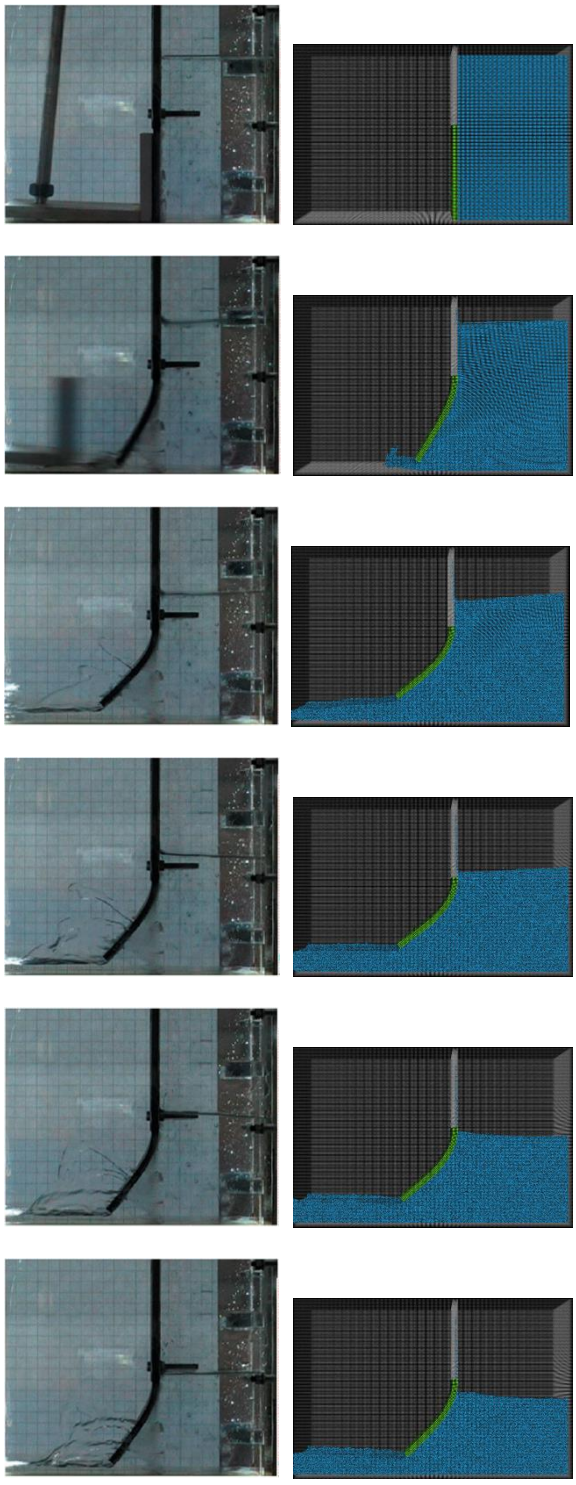

Fig. 4 Experiment and Simulation results every 0.08s

4.3. Discussion

Comparing the simulations carried out with the codes developed in this study to the experiments, the results were that the fluid was gradually flowing faster, although it was initially similar to the experiments. A graph of the displacement at which Gate has moved in the x-axis direction is shown in Fig. 5, with experimental data and other simulation data. The graph for displacement moved in the z-axis direction is shown in Fig. 6. The simulation results show that the gate initially rises similar to the test results, but goes down slower than the test results.

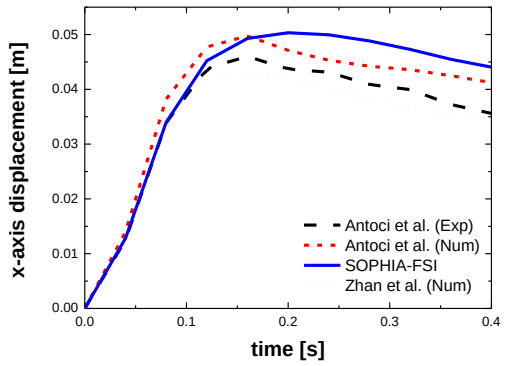

Fig. 5 The displacements at the gate's free side in the x-axis direction

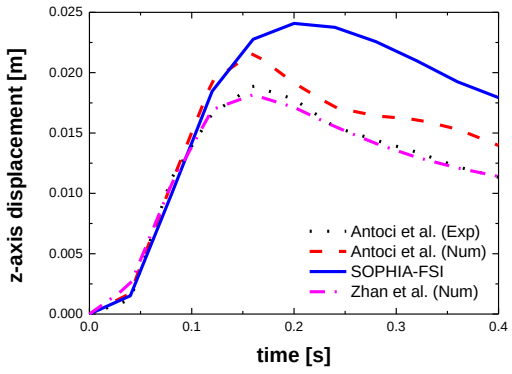

Fig. 6 The displacements at the gate's free side in the z-axis direction

When coupling the fluid and structure, the interaction force is set to pressure force, which is calculated based on density. The method for calculating density is mass summation method, which reduces density when the number of surrounding particles decreases. If the gate is highly deformed at the end of the simulation, the distance between the structure particles will become longer that the density will be reduced. Because of this, the pressure becomes smaller and the gate does not push enough fluid and goes down slowly.

5. Summary

In this study, the SOPHIA code, an 3D GPU parallelized SPH solver developed by Seoul National University, was added to the FSI. The method for calculating the elastic force of structure was added to the SOPHIA code and further coupling of fluid and structure was performed. Based on this code, Dam Breaking Through a Rubber gate experiment was simulated. Compared with the results of the experiment and other numerical simulation results, some errors occurred after the mid-term. The causes of this were estimated by calculating the density of the structure and will be improved in the future.

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