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Shock Energy Dissipation during Shock-bubble Interaction in bubbly liquid

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1. Introduction

Steam explosion is a phenomenon which can threat the integrity of the containment structure with high pressure impulses when the molten core drops into a water pool either inside or under the reactor vessel bottom during a severe accident of a light water reactor. When the molten reactor core breaches the reactor vessel bottom and spilled to a reactor cavity of a PWR where a pool of water exists due to a severe accident management measure called the wet cavity, it may cause an ex-vessel steam explosion; the condition in the containment vessel involving subcooled water at low pressure is relatively favorable to cause a steam explosion. This study is an attempt to develop a method for suppressing steam explosions such a situation. We focus on the shock wave mitigation effect by micro-bubble clouds, i.e. the shock-bubble interaction (SBI). This study started from an idea that micro-bubble clouds can buffer strength of the shock wave produced by a steam explosion.

SBI had been studied for an about half century. Campbell constructed a shock model regarding bubble mixture as homogeneous media and compared experimental data which he got from his shock tube experiments [1]. Hsieh studied sound propagating in bubbly mixture and investigated effects of heat conduction on the sound waves in particular [2]. Murray investigated disturbance at change of ratio between gas and liquid in homogeneous media [3] and Nigmatulin researched structure of shock waves which pass liquid containing gas bubbles [4]. Wijngaarden defined motion of mixture of gas and liquid theoretically and investigated characteristics of SBI such as structure of a shock and an effect of relaxation of a shock [5, 6, 7, 8]. Tan made his shock-bubbly media model consisting of an equation set about two-phase bubbly mixture and expected the motion of gas bubbles and liquid at steady shock load [9, 10, 11]. Kwak constructed a single bubble model for prediction of sonoluminescense phenomena which describes motion of a single bubble and heat transfer in, on, and around the bubble and validated his model with internal and external experimental data [12, 13].

In this study, to clarify the mechanism of shock mitigation in microbubbles, we focused on shock mitigation in underwater microbubble clouds in terms of energy dissipation. Microbubbles are expected to be the role of air cushions in liquid environment, and the present study describes the energy mechanism of shock mitigation by microbubbles. One-dimensional model of shock-bubble interaction (SBI) is developed, which contains two-phase mass and momentum equation and heat transfer model between a bubble and liquid describing the dissipation of bubble energy. The 1D SBI model is transformed into an ordinary differential equation set and used for numerical calculation by a MATLAB tool to obtain results of bubble behavior when shock waves pass the bubble mixture. After the calculation, releasing heat energy and shock damping time are calculated as factors of shock mitigation and discussed with shock strength, bubble radius, and gas volume fraction.

2. Methodology

2.1. Physical model

This study suggested SBI model which was based on the model Tan developed [9]. Therefore, this SBI model borrows several assumption applied to Tan's model. First of all, (1) each bubble remains spherical and (2) has a uniform size. (3) Bubbles do not merge together, break in smaller ones, or collapse. (4) Pressure in a bubble is uniform, which means this SBI system operates in time scale of micro second. (5) Liquid is incompressible as compressibility of liquid is much smaller than compressibility of gas. The mass equations are written as

$$\frac{\partial}{\partial t}\left(\alpha\rho_g\right)+\frac{\partial}{\partial x}\left(\alpha\rho_g u_g\right)=0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(1-\alpha\right)+\frac{\partial}{\partial x}\left[\left(1-\alpha\right)u_l\right]=0 \tag{2}$$

where α, ρ, and u denote the gas volume fraction, density, and velocity, and the subscripts g and l denote the gas and the liquid respectively. The momentum equations are written as

$$\frac{\partial}{\partial t}\left(\alpha\rho_g u_g\right)+\frac{\partial}{\partial x}\left(\alpha\rho_g {u_g}^2\right)=-\alpha\frac{\partial p_l}{\partial x}+Nf \tag{3}$$

$$\frac{\partial}{\partial t}\left[\left(1-\alpha\right)\rho_l u_l\right]+\frac{\partial}{\partial x}\left[\left(1-\alpha\right)\rho_l {u_l}^2\right]=-\left(1-\alpha\right)\frac{\partial p_l}{\partial x}-Nf \tag{4}$$

where p, N, and f denote the pressure, the number density of microbubbles, and the force which is influenced to a bubble or the liquid each other.

Bubble dynamics of a radial motion by the shock wave in the liquid is presented by Rayligh-Plesset equation. It is written as

$$\frac{p_g-p_\infty}{\rho_l}=R\frac{D^2R}{Dt^2}+\frac{3}{2}\left(\frac{DR}{Dt}\right)^2+\frac{4\mu_l}{\rho_l R}\frac{DR}{Dt}+\frac{2\sigma_l}{\rho_l R}-\frac{1}{4}\left(u_g-u_l\right)^2 \tag{5}$$

where R and μ denote the radii of the microbubble and the viscosity of the liquid. And p_∞ denotes the ambient liquid pressure with 1 bar of the constant value.

The shock wave in the bubbly environment moves with a constant speed v_s, which means the bubble behavior in the SBI system follows steady state by an observer on the surface of the shock with the constant speed. Therefore, the terms of the time parameter in the equations are removed and the equations become form of ordinary differential equation. They are rewritten as

$$\frac{d}{dx}\left(\alpha\rho_g v_g\right)=0 \tag{6}$$

$$\frac{d}{dx}\left[(1-\alpha)v_l\right]=0 \tag{7}$$

$$\frac{d}{dx}\left(\alpha\rho_g {v_g}^2\right)=-\alpha\frac{dp_l}{dx}+Nf \tag{8}$$

$$\frac{d}{dx}\left[(1-\alpha)\rho_l {v_l}^2\right]=-(1-\alpha)\frac{dp_l}{dx}-Nf \tag{9}$$

$$\begin{aligned}\frac{p_g-p_l}{\rho_l}&=R\frac{d^2R}{dx^2}+R\frac{dv_g}{dx}\frac{dR}{dx}+\frac{3}{2}{v_g}^2\left(\frac{dR}{dx}\right)^2\\&+\frac{4\mu_l}{\rho_l R}v_g\frac{dR}{dx}+\frac{2\sigma_l}{\rho_l R}-\frac{1}{4}\left(v_g-v_l\right)^2\end{aligned} \tag{10}$$

where v_g and v_l are gas and liquid velocity on the moving frame respectively.

The energy equation of the SBI model is refered to Kwak [12]. The bubble and the liquid transfer heat energy each other through a thermal boundary layer. A main mechanism of the heat transfer is conduction, written as

$$\frac{dQ}{dt}=\left(k_l\nabla T\right)A_b=k_l\left(4\pi {R_b}^2\right)\left(\frac{\partial T}{\partial r}\right)_{r=R_b} \tag{11}$$

where Q, k_l, A_b, R_b denote the transferred heat energy, conductivity of the liquid, the surface area of the bubble, and the radii of the bubble respectively. The heat energy consists of the internal energy by temperature of the bubble and the kinetic energy which is caused by expansion and contraction of the bubble. Using $U_b = m_bC_{v,b}T_b$ of the internal energy and $W_b = P_bV_b$ of the kinetic energy, the conduction equation is reorganized in terms of pressure and temperature of the bubble, written as

$$\frac{dP_b}{dt}=-\frac{3\gamma P_b}{R_b}\frac{dR_b}{dt}-\frac{6(\gamma-1)k_l\left(T_{bl}-T_\infty\right)}{R_b\delta} \tag{12}$$

$$\frac{dT_{bo}}{dt}=-\frac{3(\gamma-1)T_{bo}}{R_b}\frac{dR_b}{dt}-\frac{6(\gamma-1)T_{bo}\left(T_{bl}-T_\infty\right)}{P_bR_b\delta} \tag{13}$$

where γ, δ, T_{bo}, T_{bl} denote the heat capacity ratio of the gas bubble, the thermal boundary layer, temperature on the bubble, and temperature at the thermal boundary.

In case of liquid around a bubble, energy transfer equation is represented below.

$$\rho_l c_{p,l}\left(\frac{\partial T}{\partial t}+u\cdot\nabla T\right)=-\nabla\cdot q \tag{15}$$

where $c_{p,l}$ is specific heat capacity of liquid on constant pressure. Heat flux in liquid is expressed as thermal conduction of liquid ($q= -k_l \nabla T$). An equation describing a relation between thermal boundary layer, bubble radius and gas temperature is derived from mentioned energy equation in liquid above.

$$\begin{aligned}&\left[1+\frac{\delta}{R}+\frac{3}{10}\left(\frac{\delta}{R}\right)^2\right]\frac{d\delta}{dt}=\frac{6\alpha_l}{\delta}-\left[2\frac{\delta}{R}+\frac{1}{2}\left(\frac{\delta}{R}\right)^2\right]\frac{dR}{dt}\\&-\frac{\delta}{T_{bl}-T_\infty}\left[1+\frac{1}{2}\frac{\delta}{R}+\frac{1}{10}\left(\frac{\delta}{R}\right)^2\right]\frac{dT_{bl}}{dt}\end{aligned} \tag{16}$$

2.2. Calculation

MATLAB was used as a calculation code function “ode23” in the library of the program was used for solving the ordinary differential equation set describing the given SBI system. Initial conditions of unknowns are 1 as every variable became dimensionless. Thermal boundary layer δ wasn’t regarded as an unknown variable of the ODE set and was replaced with $0.3R$ instead.

We got results of time histories of liquid and gas pressure, bubble radius, and gas temperature after MATLAB calculation. From the results, we obtained energy of bubble clouds which is released to the liquid surrounding the bubbles by bubble oscillation. The release energy of bubble clouds consists of work that the bubble clouds do to surrounding liquid at a bubble wall and heat that is transferred outside from the bubbles.

Sets of calculation cases are introduced at Table. 1. We calculated 900 cases which have different initial conditions of the bubble radius, the gas volume fraction, and the Mach number. A representative for describing the shock strength is Mach number which is related to the ratio of the shock pressure to the ambient pressure.

Table. 1 Information of iteration calculations

Parameter	**Symbol**	**min. value**	**max. value**	**# of cases**
Bubble radius	R_0	5μm	500μm	10
Volume fraction	α_0	0.01%	10%	10
Shock strength	Ma^2	2	500	9

3. Results

Fig.1 shows the thermal dissipation energy with various sizes of the bubbles and gas volume fractions. The higher gas volume fraction dissipates more heat energy in the bubble cloud as common sense. In case of the size of the bubbles, the smaller bubble seems to be capable to dissipate heat energy more; however, the effective range of the size of bubbles is less than 10μm. It is hardly possible to make smaller bubble than the order of 10μm of the diameter. In other words, the size of bubbles is not a main factor of energy dissipation of bubbles, in terms of quantity of energy dissipation.

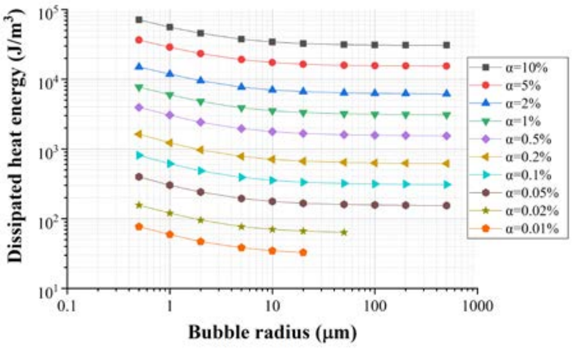

Fig.1 Volumetric thermal dissipation energy with various sizes of bubbles and gas volume fraction at *Ma* = 1.414.

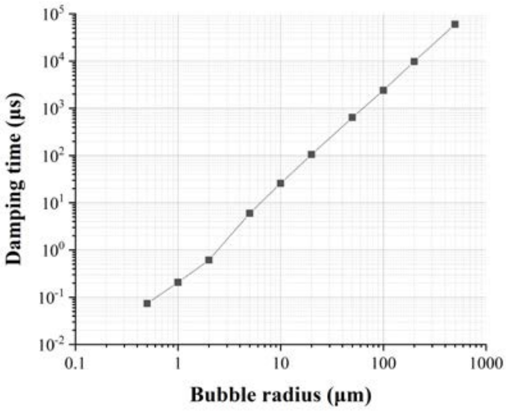

Fig.2 Total damping time until response of bubbles stops at *Ma* = 1.414 and α_0 = 1%.

Fig.2 shows the total damping time until the bubbles stop their motions as expansion and contraction. Although the graph shows only one case with volume fraction of 1%, the total damping time doesn't change with various gas volume fraction. In case of the size of bubbles, the smaller bubbles make the damping time faster, which means the shock energy is dissipated faster in the bubbly mixture with the smaller bubbles.

The results insist that the gas volume fraction in the bubbly liquid is the main factor of energy dissipation of shock wave in terms of energy quantity. The size of bubbles is minor for the quantity of energy dissipation, but it makes dissipation mechanism be faster with smaller bubbles, which can be a significant factor of energy dissipation in the SBI system.

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