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Reconstruction of Unmeasured data by Compressive Sensing for Correlation Development

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1. Introduction

The problem of prediction of thermal-hydraulic properties of nuclear reactor can be considered as an optimization problem where the parameters of correlation function are estimated from experimental data. Normally, the experiments cannot cover all of the measurements of interest. We developed a reconstruction method using compressive sensing (CS) for the unmeasured data. The least-squares method is a traditional approach to find the parameters of correlation. Method of least-squares often leads a resulting cost function to local minima and is apt to be ill-conditioned. In this paper, the method of simulated annealing (SA) is used for estimating thermal-hydraulic parameters of the empirical correlation from experimental data.[1]

2. Correlation development and compressive sensing

In this section, techniques used to generate correlation function and reconstruct unmeasured data are described.

2.1 Correlation function development [1]

A general format of the correlation function is given by

$$f(x,y,z) = a \cdot x^b \cdot y^c \cdot z^d. \tag{1}$$

where f is the output correlation function, x, y and z are input variables, and a, b, c, d are the parameters to be estimated.

For the demonstration of the developed method, we select the correlation of degradation factor F of heat transfer coefficient given in [2]. The degradation factor F is introduced to connect the wall condensation heat transfer with the noncondensing gases. This factor can be expressed in the function of the parameters as follows:

$$F = a \cdot W_{air}^b \cdot Ja^c \cdot Re_f^d \tag{2}$$

where W_{air} : air mass fraction, Ja : Jakob number, Re_f : liquid film Reynolds number. The parameters and a, b, c, d in are determined with experimental data.

2.2 Compressive sensing

CS is a signal processing technique to reconstruct a signal from far fewer measurements than required by the Shannon-Nyquist information criterion. We can get successful reconstruction, although the signal is sparse, which means most of the elements of the frequency domain signal are zero or negligible.[3] The simple form of CS can be described as

$$y = Ax . \tag{3}$$

In this equation, the N-dimensional sequence, y is called measurement vector and formed by encoding frequency domain signal x into an M-dimensional measurements through a linear transformation by the $M \times N$ measurement matrix A (where $m < n$). The vector x is a discrete signal and called k-sparse if x has at most $k \ll N$ nonzero entries. CS aims to reconstruct a signal x called k-sparse from $y = Ax$ by solving the following ℓ_0-minimization:

$$\min\{\|x\|_0 : Ax = y\}. \tag{4}$$

This ℓ_0-minimization is a combinatorial optimization problem and is considered as NP-hard. On the other hand, and ℓ_2-minimization (least-squares method) is a viable method. But it is known cannot find the sparse solution. Hence, ℓ_0 and ℓ_2-minimization are replaced by the ℓ_1-minimization

$$\min\{\|x\|_1 : Ax = y\}. \tag{5}$$

With partitioning the measured and lost signals, equation (3) can be regrouped:

$$Ax *= y \Rightarrow Ax *= b, Bx *= u. \tag{6}$$

where b is known, u is unmeasured vector, and $x*$ is the optimal solution in the frequency domain. [4]

2.3 Simulated annealing [1]

SA imitates the idea of annealing in metallurgy containing heating and controlled cooling of material to form crystalline structure with minimum energy. We can simulate the slow process of cooling to find the global optimal solution. The advantage of SA is that it can reduce the probability of the solution being captured at local minima. SA can approximately find the global minimum. SA algorithm starts with a randomly generated initial solution X at the initial temperature T and generates the objective function. Then, it generates a new random candidate solution vector, Y, in the neighborhood of current solution vector X. It judges whether to accept new solution. If the new solution vector, Y, is better, the algorithm accepts it and updates the current solution. Otherwise, the algorithm accepts Y the probability: $p(\Delta E) = \exp(-\Delta E/T)$ based on Boltzmann probability density. It decreases the temperature stepwise, and decides whether the minimum temperature is reached. It repeats this process until the stopping criterion is met.

To develop the correlation function, we try to find the optimal solution parameters a, b, c, d in Equation (2) with the optimization problem:

$$\min_{a,b,c,d} \left\| a \cdot W_{air}^{b} \cdot Ja^{c} \cdot Re_{f}^{d} \right\|_2 \qquad (7)$$

3. Demonstration

The experimental data set is presented in Fig. 1. It consists of three input data (a), (b), (c), and the output function (d). Some data set of steps 75 to 100 are removed intentionally, and that part is considered unmeasured data. We reconstructed them by using CS, and the result is shown in Fig. 2. Next, we applied SA to the data set involving reconstructed data to create optimal correlation.

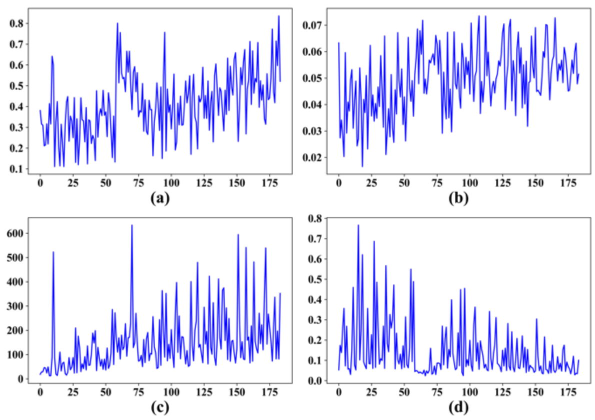

Fig. 1. Measured data by experiment (a) air mass fraction, (b) Jakob number, (c) Reynolds number and (d) degradation factor

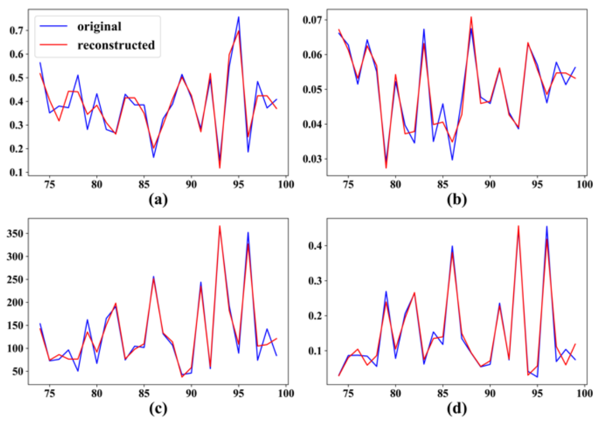

Fig. 2. Reconstructed data set of steps 75 to 100

Fig. 3 illustrates the accuracy of the developed correlation using the reconstructed data. Red bullets mean the correlation generated with lost data included. The reconstructed data gives an accuracy similar with original data. This means that the measured data is reconstructed with sufficient accuracy. In this figure, we can find the SA gives more accurate result than the previous least-squares result given in [2].

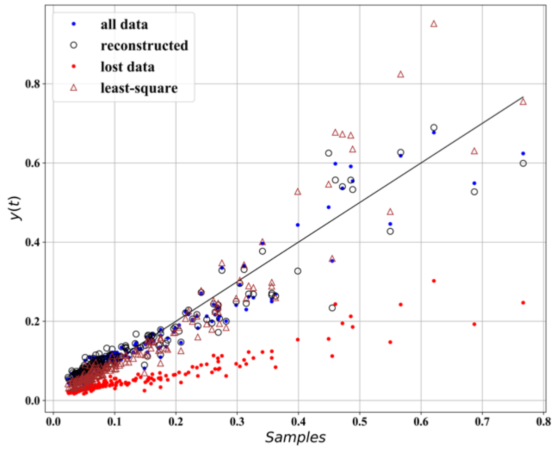

Fig. 3. Accuracy of the correlation with reconstructed data

4. Conclusions

This paper describes the process of reconstruction of unmeasured data using CS to determine the parameters of empirical correlation. Although the least-squares method is generally applied, the parameters were determined using the SA to overcome the ill-posedness of least-squares method. This can be seen clearly in the results of accuracy of correlation. The CS method provides accurate and robust solution similar to that of generated with no-lost data. The proposed method can be applied to many types of correlation development by expanding the data set or incorporating unmeasured data domain.

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