Modeling of the wake-induced lift force acting on an unbounded bubble at arbitrary Reynolds number

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1. Introduction

In the safety analysis of a nuclear reactor, the Eulerian two-fluid method with interfacial momentum transfer models has been widely utilized due to its computational efficiency compared to the method that fully resolves the complex interfaces of the two-phase mixture. However, current models of the lift, wall-lift, and turbulent dispersion force experienced by bubbles are not sufficiently universal to predict the lateral void fraction distribution at high-*Re* condition [1], where *Re* is bubble Reynolds number $Re = \rho U_R d/\mu$, d is the volume equivalent diameter of the bubble ($V = \pi d^3/6$, V is the volume of the bubble), $U_R = |\boldsymbol{U_R}| = |\boldsymbol{U_B} - \boldsymbol{U_L}|$ is the magnitude of the bubble's relative velocity ($\boldsymbol{U_B}$ is the bubble velocity, and $\boldsymbol{U_L}$ is the velocity of the undisturbed liquid flow taken at the bubble center), and ρ and μ are the density and dynamic viscosity of the liquid, respectively. With this background, Lee and Lee [2] reported experimental measurement results of the lift force coefficient C_L at $440 < Re < 7200$. Lee and Lee [2] also suggested a physical model that can be used to effectively estimate C_L at high-*Re*.

In this abstract, the C_L model of Lee and Lee [2] is derived from Eq. (3.14) of Magnaudet [3] to illustrate a general picture of the wake-induced dynamics experienced by single bubbles in both unbounded and bounded cases. Moreover, the C_L model's prediction results are compared with both experimental and numerical data of the C_L reported by Lee [4].

2. Derivation of the wake-induced lift force

The force acting on a body moving in a fluid at rest with a fixed shape $\boldsymbol{F}_H$ can be expressed by Eq. (3.14) of Magnaudet [3], which is given as follows.

$$\frac{\boldsymbol{e}_T \cdot \boldsymbol{F}_H}{\rho} = -\boldsymbol{e}_T \cdot \left(\frac{d_\Omega \left(\boldsymbol{A} \cdot \boldsymbol{U}_B \right)}{dt} + \boldsymbol{\Omega} \times \left(\boldsymbol{A} \cdot \boldsymbol{U}_R \right) \right)$$
$$+ \int_{V_L} \left\{ \left(\boldsymbol{\omega} + \boldsymbol{\omega}_B \right) \times \boldsymbol{U}_L \right\}_0 \cdot \left(\boldsymbol{U}_{B,A} - \boldsymbol{U}_{L,A} \right) dV \quad (1)$$
$$- \frac{2}{Re} \int_{S_B} \left(\boldsymbol{U}_{L,A} - \boldsymbol{U}_{B,A} \right) \times \left(\boldsymbol{\omega} - 2\boldsymbol{\Omega} \right) \cdot \boldsymbol{n} \, dS$$

where $\boldsymbol{U_{B,A}}$ and $\boldsymbol{U_{L,A}}$ are the auxiliary unit velocity of the body and the auxiliary irrotational velocity field, respectively. $\boldsymbol{\omega}$ and $\boldsymbol{\omega_B}$ are the free vorticity and bound vorticity, respectively. $\boldsymbol{\omega_B}$ only exists at the surface of the bubble. $\boldsymbol{A}$ is the added mass tensor, $\boldsymbol{n}$ is the unit normal vector at the bubble's interface, S_B indicates bubble's surface, V_L indicates volume of the liquid outside of the bubble, $\boldsymbol{\Omega}$ is the angular velocity vector of the body, and $\boldsymbol{e_T}$ is the unit vector that is the translational component of $\boldsymbol{U_{B,A}}$. In comparison with Eq. (3.14) of Magnaudet [3], all quantities in Eq. (1) are dimensional ones, and the contribution from the outside wall is ignored by assuming single bubbles sufficiently far from a wall. Closed terms in the right-hand side of Eq. (1) is related to the inertia, and the surface integral is mainly related to the viscous contribution of the drag force. We are interested in the wake-induced force $\boldsymbol{F_{wake}}$ that results in the horizontal translational motion of a bubble.

$$\boldsymbol{e}_T \cdot \boldsymbol{F}_{wake} = \rho \int_{V_L} \left\{ \left(\boldsymbol{\omega} + \boldsymbol{\omega}_B \right) \times \boldsymbol{U}_L \right\} \cdot \left(\boldsymbol{e}_T - \boldsymbol{U}_{L,A} \right) dV \quad (2)$$

Fig. 1 shows the present idealized concept of the wake-induced-zigzag motion of a free rising bubble in a two-dimensional viewpoint. Assuming instantaneous planar symmetry of the wake, flow around the bubble near the symmetry plane can be approximated to such circumstances. The thick dashed line indicates the vortex-ring like vortex structure at the bubble equator. The cross point and dot point indicate vorticity vectors at the *xy*-plane directing to the positive *z*-direction and the negative z-direction, respectively.

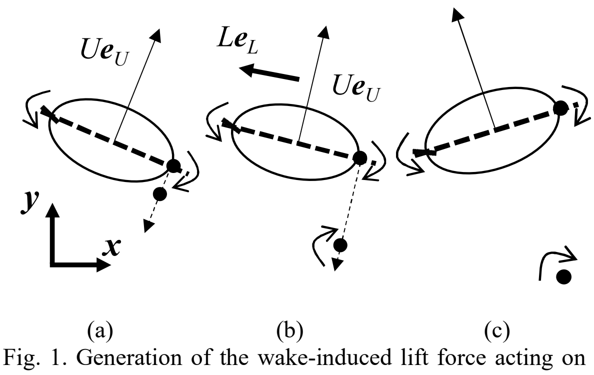

Fig. 1. Generation of the wake-induced lift force acting on a zigzagging bubble

By approximating the vortex at the surface of the bubble to axisymmetric, the contribution of $\boldsymbol{\omega_B}$ to lift force can be ignored. When the wake becomes unstable, a hairpin vortex is detached from one side, as illustrated in Fig. 1. The vortex detached from the right side of the bubble in Fig. 1 can be approximately expressed as the negative z directional line vortex $\Gamma\delta(x\text{-}x_V)\delta(y\text{-}y_V)(\text{-}\boldsymbol{k})$, where Γ is the magnitude of the circulation, (x_V, y_V) is the position of the detached vortex, and $\delta(x)$ is the Dirac delta function.

In Eq. (2), $\boldsymbol{U_L}$ is the liquid velocity at (x_V, y_V) and represents the velocity of the convected vortex. $\boldsymbol{U_L}$ (x_V, y_V) may be similar to the body's relative velocity right after the detachment of the vortex and will be continuously slowed down. $\boldsymbol{U_{L,A}}$ is determined by the kinematical boundary condition at the bubble interface, $\boldsymbol{U_{L,A}} \cdot \boldsymbol{n} = \boldsymbol{e_T} \cdot \boldsymbol{n}$, and $|\boldsymbol{U_{L,A}}|$ approaches to 0 at far from the body, i.e., $\boldsymbol{U_{L,A}}$ decays as r^{-3}. Therefore, it can be approximated that $\boldsymbol{U_L} \approx U_R\boldsymbol{e_V}$ and $\boldsymbol{U_{L,A}} << 1$ at (x_V, y_V), where $\boldsymbol{e_V}$ is the unit vector directed to the vortex convection. Then Eq. (2) can be simplified as follows.

$$\begin{aligned} \boldsymbol{e}_T \cdot \boldsymbol{F}_{wake} &\approx \rho \int_{V_L} \{\Gamma\delta(x)\delta(y)(-\boldsymbol{k})\times(U_R\boldsymbol{e}_V)\}\cdot \boldsymbol{e}_T dV \\ &= \rho\Gamma U_R \left[\int_l dz\right](-\boldsymbol{k}\times\boldsymbol{e}_V)\cdot\boldsymbol{e}_T \end{aligned} \tag{3}$$

If the line integral is equal to l and $\boldsymbol{e_V} = -\boldsymbol{U_R}/|\boldsymbol{U_R}|$, $F_{L,wake}$ becomes $\rho l\Gamma U_R$ also derived by de Vries et al. [5] in the case of a free rising bubble.

By approximating the vorticity at the center of the viscous vortex detached from the bubble to the vorticity at the bubble's equator ω_E, Γ can be approximated to $4\pi(\mu/\rho)t_V\omega_E$ [2, 4], where t_V is the vortex shedding period. In the case of clean bubbles, Veldhuis et al. [7] experimentally showed that the t_V is the same as $1/f_{(2,0)}$, where $f_{(2,0)}$ is the (2,0) mode frequency of the bubble shape oscillation.

$$f_{(2,0)} = \frac{1}{2\pi}\left(\frac{16\sqrt{2}\chi^2}{\left(\chi^2+1\right)^{3/2}}\right)^{1/2}\left(\frac{\sigma}{\rho r^3}\right)^{1/2} \tag{4}$$

where $\chi = b/a$ is the shape aspect ratio, a and b are the lengths of the minor and major semi-axes of the spheroidal shape of the bubble, respectively. σ is the liquid surface tension, and r is $d/2$. Until this stage, the derivation of $F_{L,wake}$ is applicable to unbounded single bubbles at arbitrary *Re* conditions.

3. Comparison with data (unbounded single high-*Re* bubbles in a linear shear flow)

Accounting for the spatial variation of ΓU_R in the case of single unbounded bubbles in a linear shear flow, the time-averaged lift force acting on the bubble can be approximated to $4\rho l\Gamma_0\omega_0 X$. Γ_0 is Γ evaluated with U_R at the center of the bubble, ω_0 is the shear ratio of the flow, and X is the horizontal distance from the bubble center to consider the location of the detached vortices from the bubble [2]. At high-*Re* condition, ω_E can be derived from the potential theory as follows [6].

$$\omega_E = \frac{U_R}{r}\frac{2\chi^{5/3}\left(\chi^2-1\right)^{3/2}}{\chi^2\sec^{-1}\chi-\left(\chi^2-1\right)^{1/2}} \tag{5}$$

Applying Eqs. (4) and (5) to $4\rho l\Gamma_0\omega_0 X$ with $l = 2b$ and $X = b/2$, Lee and Lee [2] finally derived the wake-induced contribution of the lift coefficient $C_{L,wake}$ as follows.

$$C_{L,wake} = -\frac{24\pi}{2^{3/4}}\frac{\chi^{4/3}\left(1+\chi^2\right)^{3/4}\left(\chi^2-1\right)^{3/2}}{\chi^2\sec^{-1}\chi-\left(\chi^2-1\right)^{1/2}}Oh \tag{6}$$

where Oh is Ohnesorge number $Oh = \mu/(\rho\sigma d)^{1/2}$. The miswritten constant of proportionality of [2] is corrected here. In order to describe the positive C_L at intermediate *Re*, Lee and Lee [2] added 0.5 that is the added mass coefficient of spherical body to $C_{L,wake}$ based on the good agreement between the model and their data.

$$C_L = 0.5 + C_{L,wake} \tag{7}$$

Fig. 2 shows the prediction results of Eq. (7) by curves in the case of single air bubbles in water flow at $d > 1$ mm. Both results obtained by approximating quasi-steady rising of bubbles in experimental cases [2] and numerical cases [4] are also compared with them. The experimental data were obtained by using contaminated bubbles at 26.7°C ($440 < Re < 7200$), and the numerical data were obtained from clean bubbles at 29.9°C ($400 < Re < 4000$). For χ, $(1 + 0.21Eo^{0.58})$ and $(1.8 + 0.036\ Eo^{1.1})$ are used for the experimental case, and numerical case, respectively [2, 4], where Eo is Eötvös number $Eo = \rho g d^2/\sigma$, and g is the gravitational acceleration. At $d < 4$ mm, $l = b/2$ is applied to represent experimental observation [5].

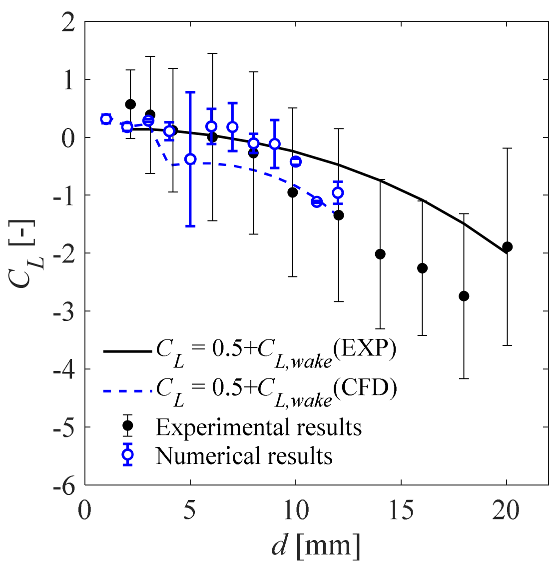

Fig. 2. Validation of the model on the wake-induced contribution to the lift coefficient at high-*Re* condition [2, 4]

As shown in Fig. 2, good agreement between the model, numerical results, and experimental data can be seen. Lee and Lee [2] also showed that Eq. (7) could also be applicable to unbounded single bubbles in a linear shear flow at $4 < Re < 40$, with proper modification of ω_E and l. The validity of Eq. (7) at both *Re* of O(10) and O(10^3) shows the characteristic of Eq. (1) that available at arbitrary *Re* conditions.

Based on these agreements, our next study will be focused on the generalization of the current approach to the cases of *Re* of O(10^2) and wall-bounded bubbles. For the accurate simulation of various gas-liquid two-phase flows that occurred in the safety analysis of a nuclear reactor by using the Eulerian two-fluid approach, the model of turbulent dispersion force also should be improved. It is hoped that our new modeling of the vortices detached from deformed bubbles can contribute to the improvements of these subgrid-scale models.

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