Extension of Pin-based Point-wise Energy Slowing-down Method into Double Heterogeneity Fuel

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1. Introduction

Recently, Pin-based pointwise energy Slowing-down Method (PSM) [1] was proposed for overcoming the limitation of the equivalence theory [2] in the resonance treatment. Also, it was successfully implemented into DeCART [3] and verified with some high temperature gas reactor (HTGR) problems [4].

However, the method was developed with the assumption that a fuel region is homogeneous. Thus, it is not directly applied to the double heterogeneity (DH) fuel of HTGR which consists of a matrix and TRISO fuel particles. In our previous work, the PSM was just applied to fuel pins with a homogeneous fuel compact.

For applying the PSM into a DH fuel, it is necessary to homogenize the fuel compact region. This approach is similar to the two-step resonance treatment method for the DH fuel of SCALE code system [5]. In the first step, pointwise homogenized cross sections for the compact region should be prepared after solving the slowing down equation for a spherical unit cell composed of a TRISO and a matrix layer. The shielded cross section for the homogenized fuel compact can be then obtained using the conventional equivalence theory. However, the verification report [6] shows that the spherical unit cell without a moderator material causes the error over 300 pcm. Also, the method has the limitation of the equivalence theory.

In this study, a modified two-step method based on the PSM, named as PSM-DH, is proposed for the resonance treatment in a DH fuel. The method has an improved spherical unit cell model with a moderator in the outermost layer and applies the PSM in the self-shield cross section generation step for the homogenized compact region.

2. Methodology

The simple review of the PSM was described in section 2.1 and then the extension of the PSM for a DH region was summarized in the next section.

2.1. Review of PSM

The PSM solves the slowing down equation for all energy points with the collision probability (CP) as follows:

$$\Sigma_{t,i}\phi_i V_i = \sum_{j\in F} P_{ji} V_j Q_{s,j} + P_{Mi,g} V_M Q_{s,M} \quad (1)$$

where i and j are fuel sub-region indexes and M is a moderator region index. P_{ij} is the CP which a neutron born at a sub-region i has its first collision at sub-region j. V_i is the volume of the sub-region i and $Q_{s,j}$ is the slowing down scattering source of the sub-region j.

Firstly, collision probability tables are prepared using the method of characteristics (MOC). For the CP of an isolated fuel pin, the fuel region is divided into about 15 sub-regions and fixed source problems are solved by MOC for every fuel sub-region and about a few hundred levels of total cross section range of interest as follows:

$$\Omega\cdot\nabla\psi(r,\Omega) + \Sigma_{t,i}^{T}(r)\psi(r,\Omega) = \frac{1}{4\pi}\delta_i(r)\,, \quad (2)$$

$$P_{ij}^{iso}(\Sigma_{t,i}^{T}) = \frac{\Sigma_{t,j}^{T}\phi_j V_j}{V_i}\,, \quad (3)$$

where $\Sigma_{t,i}^{T}$ is a total cross section in sub-region i. P_{ij}^{iso} is the CP in the isolated fuel pin.

The CP for a lattice pin can be obtained from the CP of the isolated pin and the shadowing effect factor. The factor can be calculated using the Carlvik's two-term rational approximation [2] and Dancoff factor [2] by the enhanced neutron current method. The Dancoff factor is obtained by solving the fixed source problem with MOC.

The fuel escape probability for the isolated and lattice system, $\overline{P}_{e,F}^{iso}$ and $\overline{P}_{e,F}$, can be calculated from the Carlvik's two-term rational approximation and the shadowing effect factor for the fuel region can be obtained as the following equation.

$$\eta_F = \frac{\overline{P}_{e,F}}{\overline{P}_{e,F}^{iso}} \quad (4)$$

Thus, the CP inside the fuel sub-region is expressed as follows:

$$P_{ij} = P_{ij}^{iso}\frac{1-P_{e,i}}{1-P_{e,i}^{iso}}\,, \quad (5)$$

where

$$P_{e,i} = P_{iM} \approx \eta_F(u)P_{e,i}^{iso}\,, \quad (7)$$

$$P_{e,i}^{iso} = P_{iM}^{iso} \approx 1-\sum_{j\in F} P_{ij}^{iso}(\Sigma_{t,F}^{T})\,, \quad (8)$$

and P_{iM}^{iso} and P_{iM} are the CP from the fuel region to the non-fuel region in the isolated and lattice system, respectively. Also, the CP from the non-fuel region can be readily obtained from the above equations.

Finally, the flux at an energy point of the slowing down equation, Eq.(1), can be calculated with the CP.

The further information for the PSM can be found in the reference [1].

2.2. Pointwise Energy Slowing Down Method for a DH region

Prior to the PSM procedure, it is necessary to obtain the pointwise homogenized cross section for the fuel compact. For this, a spherical unit cell model is proposed as shown in Figure 1.

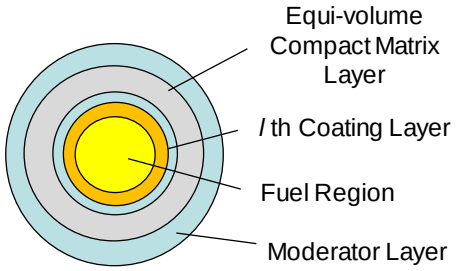

Fig. 1. A spherical unit cell for a fuel compact

Contrary to the SCALE model, the unit cell has the moderator material in the outermost layer for reflecting the effect of the moderator in a pin cell.

The radius of the graphite matrix layer can be easily calculated as follows:

$$R_{TRISO+Matrix} = \frac{R_T}{\sqrt[3]{V_f}}, \quad (9)$$

where V_f is the packing fraction of TRISO particles in the compact and R_T is the radius of a TRISO.

In addition, the radius of the moderator layer can be analytically derived from the relation [2] with the Dancoff factor as followings:

$$1-D = \frac{\exp(-\Sigma_{t,m} l_m)}{\Sigma_{t,m} \cdot (\bar{l} - l_m) + 1}, \quad (10)$$

where D is Dancoff factor and $\bar{l}$ is the mean chord length of the moderator region. l_m is the minimum distance between two compacts and $\Sigma_{t,m}$ is the total cross section in the moderator.

After replacing the chord length with $\frac{4V_m}{2\pi R_c}$ in Eq. (10), the moderator volume can be determined as follows:

$$V_m = \frac{\pi R_c}{2}\left\{\left[\frac{\exp(-\Sigma_{t,m} l_m)}{1-D} - 1\right]\frac{1}{\Sigma_{t,m}} + l_m\right\}, \quad (11)$$

where R_c is the radius of the compact.

Then, the outer radius of the moderator layer can be calculated as follows:

$$R_{TRISO+Matrix+Moderator} = \sqrt[3]{\frac{3}{4\pi N_T}\left(\pi R_c^2 + V_m\right)}, \quad (12)$$

where N_T is the number of the TRISO particles in the fuel compact.

Therefore, the pointwise energy slowing down equation for the spherical unit cell can be defined as follows:

$$\Sigma_l \phi_l V_l = \sum_{k=1}^{L} P_{k,l} V_k Q_{s,k} \quad (13)$$

where l is a layer index of a spherical unit. The collision probability for the layer of the spherical unit cell, $P_{k,l}$, can be calculated using Kavenoky technique [7].

Eq.(13) can be readily rewritten using the reciprocity relation as follows:

$$\phi_l = \frac{\sum_k Q_{s,k} \frac{P_{l,k} V_l \Sigma_l}{\Sigma_k}}{\Sigma_l V_l} = \sum_k Q_{s,k} \frac{P_{l,k}}{\Sigma_k} \quad (14)$$

After obtaining the pointwise energy flux for all layers from Eq.(14), the pointwise homogenized cross sections for the compact can be calculated.

2.3. Procedure of PSM-DH

If obtaining the pointwise homogenized cross sections for the compact, the self-shielded multi-group cross section can be generated applying the PSM as described in the section 2.1. DeCART can, then, directly use the cross section for a transport calculation in a DH region. Figure 2 shows the procedure of the PSM-DH.

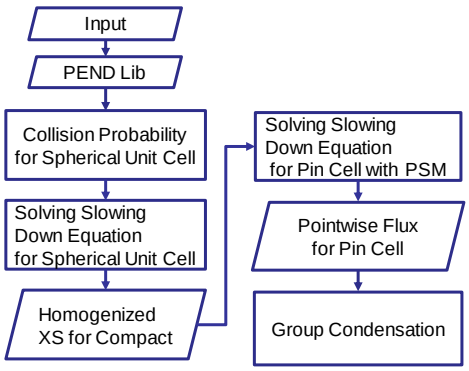

Fig. 2. Procedure of PSM-DH for the resonance treatment of a DH fuel.

3. Numerical Results

For verifying the performance of the PSM-DH, the calculation results for typical HTGR problems with DH regions were compared to McCARD [8] results.

Figure 3 and Table I shows the configuration and material number densities, respectively, of the fuel pin with DH region used in the verification calculation.

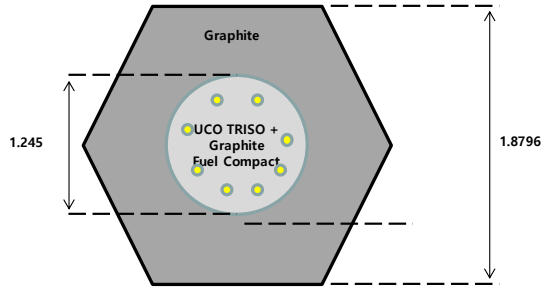

Fig. 3. Configuration of the fuel pin with DH region

Table I: Number density for the pin problem

Material		Nuclide	Number Density (#/barn-cm)
TRISO Fuel Particle	Kernel	U-235	3.70e-03
		U-238	1.99e-02
		O-16	3.55e-02
		Graphite	1.18e-02
	Porous Carbon	Graphite	5.02e-02
	IPyC	Graphite	9.53e-02
	SiC	Si-28	4.43e-02
		Si-29	2.25e-03
		Si-30	1.49e-03
		Graphite	4.81e-02
	OPyC	Graphite	9.53e-02
Compact Matrix		Graphite	8.27e-02
Block Graphite		Graphite	9.28e-02

Table II shows the comparison results between McCARD and PSM-DH/DeCART according to various packing fractions (PF) under the hot full power state (1200K). The differences in the all cases except 10% PF are under 110 pcm. If considering the statistical uncertainty caused by the random distribution of the TRISO particles in McCARD code, about 100 pcm, it is clear that they are in good agreement. In the 10% PF case, because the CP calculation for the spherical unit cell uses the white boundary condition, it seems that the very small PF causes more discrepancy. It can explain the trend related to the difference versus PF in Table II.

Table III and IV show the k_{inf} at the various temperatures with 15% and 35% PF, respectively. The maximum difference is 218 pcm at 900K. If considering the random distribution of McCARD, it can be seen that it is a reasonable difference.

Table V and VI show the comparison results for the MHTGR single block with 210 fuel pins and the small size block with 12 fuel pins, respectively. They reveal that the differences of the all cases are under 140 pcm. In the small size block, there is a rapid change of the moderation effect between the outmost fuel pin and the center fuel pin. The results show that the effect is well considered in the block.

Table II: k_{inf} of pin problem with various PFs

Packing Fraction	HFP (1200K)		
	McCARD (σ≈14pcm)	PSM-DH /DeCART	Diff.(P-M) (pcm)
10%	1.58786	1.59007	221
15%	1.49231	1.49328	97
20%	1.41349	1.41354	5
25%	1.34959	1.34866	-93
30%	1.29635	1.29562	-73
35%	1.25293	1.25186	-107
40%	1.21621	1.21550	-72

Table III: k_{inf} of pin problem with various temperatures

Temp.	PF=15%		
	McCARD (σ≈14pcm)	PSM-DH /DeCART	Diff.(P-M) (pcm)
300K	1.59333	1.59217	-116
600K	1.55027	1.54930	-97
900K	1.51790	1.51724	-66
1200K	1.49231	1.49328	97

Table IV: k_{inf} of pin problem with various temperatures

Temp.	PF=35%		
	McCARD (σ≈14pcm)	PSM-DH /DeCART	Diff.(P-M) (pcm)
300K	1.37090	1.36935	-155
600K	1.31997	1.31843	-155
900K	1.28296	1.28079	-218
1200K	1.25293	1.25186	-107

Table V: k_{inf} of MHTGR single block

Packing Fraction	PF=35%, HFP (1200K)		
	McCARD (σ≈14pcm)	PSM-DH /DeCART	Diff.(P-M) (pcm)
15%	1.59277	1.59234	-43
25%	1.47800	1.47782	-18
35%	1.39184	1.39109	-75

Table VI: k_{inf} of small size block

Packing Fraction	PF=35%, HFP (1200K)		
	McCARD (σ≈14pcm)	PSM-DH /DeCART	Diff.(P-M) (pcm)
15%	1.65387	1.65527	140
25%	1.56707	1.56770	63

35%	1.49441	1.49474	33

4. Conclusions

In this study, the extension of the PSM was proposed for performing the resonance treatment of a DH fuel of HTGR and the verification calculation results for a typical fuel pin and single fuel block problem were presented.

The pointwise homogenized cross sections for the compact was obtained after solving the slowing down equation for a spherical unit cell composed of a TRISO, a matrix layer, and a moderator layer. The moderator volume was analytically derived using the relation of the Dancoff factor and the mean chord length. Then, the shielded cross section for the homogenized fuel compact was obtained using the original PSM.

The verification calculations with the fuel pin and the single fuel block show that the proposed method has a good performance.

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