

## Validation of the Multi-Group Pin Homogenized SP3 Code SPHINCS through BEAVRS Benchmark Analyses

Jorge Gonzalez-Amoros, Hyunsik Hong, Hyun Ho Cho and Han Gyu Joo\*

Department of Nuclear Engineering, Seoul National University

1 Gwanak-ro, Gwanak-gu, Seoul, 08826, Korea

\*Corresponding author: [joohan@snu.ac.kr](mailto:joohan@snu.ac.kr)

### 1. Introduction

The achieved high computing capabilities have eased the computational burden of Direct Whole Core (DWC) calculations [[1]]. However, the computational resources required for DWC generalization for core design and analysis purposes are still limited and the two-step method with spatial and energy refinement is still essential in industry practical applications.

Despite the remarkably low computing costs that the two-step method offers [[2]], conventional assembly based 2 energy group diffusion codes suffer the difficulty of incorporating the actual core spectrum in the assembly homogenized group constants (GCs) [[3],[4]]. Due to this inherent incapability, the two-step method yields nontrivial errors, especially for highly heterogeneous problems.

In order to overcome these difficulties pin based codes employing pin homogenized multi-group (MG) GCs have been under development in recent times [[5]]. Additionally, the use of the conventional diffusion theory is being replaced by the simplified Pn method. Particularly the SP3 equations have shown a higher capability for problems with large spatial neutron flux variations such as MOX fuel loaded or rod insertion cases. Besides, they have been demonstrated to be an optimal alternative for its simple formulation and its small computational burden.

As a first phase of the two-step core analysis, the pin heterogeneous structure inside the single fuel assembly (SA) is homogenized into pin-wise GCs. Unlike in the traditional assembly-wise homogenization, in the pin-wise case the neutron balance is not preserved in each pin boundary as the spatial homogenization is performed. Due to this scale reduction, the SuPer Homogenization (SPH) method or Discontinuity Factors (DFs) are employed with the aim at reducing the pin-homogenization error [[6],[7]].

In general terms these codes applying homogenization error correction methods show good results when compared with transport reference solutions. In particular, they can reproduce the exact same results for lattice problems. However checkerboard (CB) and core problems, in spite of showing good reactivity results and intra-assembly pin power distributions, they fail at reproducing the pin power at the assembly interfaces. These solution biases have been shown to be mainly caused by the difference in the spectrum between the SA used for group condensation and the actual core environment. To improve the solution accuracy, proper

correction of the leakage effects that cause the spectral differences is strongly required.

In this regard, the pin-wise Leakage Feedback Method (LFM) has been proved to conveniently alleviate the aforementioned spectrum difference. The pin-wise LFM employs three group (3G) Leakage-To-Removal Ratios (LTRRs) which represent the fast, intermediate and thermal energy ranges independent of the actual number of groups employed in the core calculation [[8],[9]].

This being said, the NTRACER/SPHINCS two-step calculation system that involves the multi-group, pin-by-pin, SP3 SPHINCS (Simplified P3 Pin Homogenized Innovative Neutronic Core Simulator) code is developed at Seoul National University [[10]].

The finite difference method (FDM) and the SPH factors are introduced in SPHINCS to account for the errors associated with the use of pin size FDM as well as cell homogenization. The pin homogenized MG group constants are generated by SA NTRACER (Method Of Characteristics based transport code) calculations. In order to correct the leakage effect in pin homogenized cross sections in the core calculation, the 3-group pin-wise Leakage Feedback Method (LFM) is applied in SPHINCS.

For the validation of the NTRACER/SPHINCS code system and the associated calculation methodology, the BEAVRS (Benchmark for Evaluation and Validation of Reactor Simulation) core is solved and the results are compared with the DWC results obtained with NTRACER for the whole core geometry. With this purpose, this paper intends to give a detailed description of the validation process followed as well as a more thorough insight of the SPHINCS code and the calculation methods that it employs.

### 2. 2D BEAVRS NTRACER radial model

The reactor core represented in the BEAVRS benchmark is a four loop Westinghouse Pressurized Water Reactor (PWR) loaded with 193 fuel assemblies (FAs) with a 17×17 lattice array for a thermal power of 3411 MWth. The benchmark specification [[11]] provides all the detailed geometrical data and the material compositions for the major core components including the assemblies, baffle and the barrel.

The first step of the benchmark to be addressed is the modelling of BEAVRS in NTRACER, as it is employed to generate the pin homogenized GCs and the reference

solution for a later comparison. As a thorough description of the NTRACER model is already provided in [[12]], here only a very limited version of the most relevant and basic details is given.

The 193 fuel assemblies are loaded in a  $15 \times 15$  array in the core. A one-fourth of this core is modeled in a quarter core (QC) model that arranges the fuel assemblies according to the loading pattern along with the other constituents of the core: the baffle, barrel, neutron shielding panel and vessel. As shown in the subfigure on the left (Vessel) of Fig. 1 Fig. 1. BEAVRS periphery models for sensitivity analysis., the baffle is explicitly modeled because it faces the fuel assemblies, but the other structures, however, are represented by using square cells of the pin cell pitch size.

### 2.A BEAVRS simplification for GCs generation

Since the neutron flux decreases rapidly in the radial reflector region, the modeling of the barrel, the neutron shielding panel is approximated and the vessel is even omitted in the conventional neutronics calculations. On top of this, as the reflector GCs are generated by using the configurations in Fig. 2, if the outermost parts of the barrel are omitted the radial reflector GCs generation is dramatically simplified.

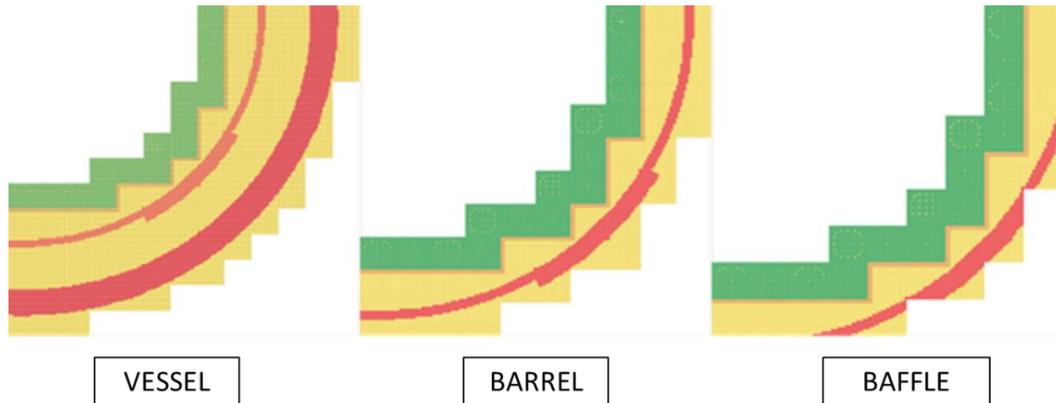


Fig. 1. BEAVRS periphery models for sensitivity analysis.

Table I: BEAVRS 2D core sensitivity analysis with NTRACER.

Case	keff (error-pcm)	Pin Power Relative Error (%)	
		RMS	MAX
VESSEL	1.00409 (-)	-	-
BARREL	1.00400 (-9)	0.01	0.02
BAFFLE	1.00400 (-9)	0.01	0.15

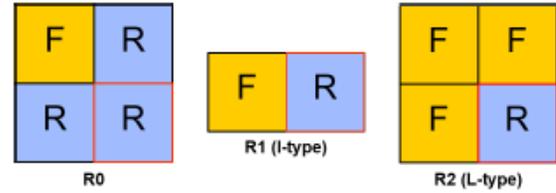


Fig. 2. Fuel-reflector configurations for reflector GCs generation.

Thus, the three models shown in Fig. 1 are considered for a sensitivity analysis in NTRACER to conclude if the peripheral parts of the barrel can be left out for subsequent calculations. The VESSEL one considers all the elements in the core and is taken as the reference, the BARREL model neglects the external vessel and the BAFFLE one only maintains the barrel portions nearest to the baffle along with the neutron shield.

Table I shows how the simplest model (BAFFLE) can be employed over the BARREL one without major implications in terms of reactivity and pin power distribution differences. As mentioned above this fact simplifies the modelling as well as the GCs generation in NTRACER.

### 3. The SPHINCS code

SPHINCS is a pin-by-pin FDM SP3 code. As previously exposed, the code presents some inaccuracies arising from the geometry and energy refinement of the GCs as well as the methodology followed in their generation and they require to be corrected.

In the subsequent subsections the corrective methods employed in SPHINCS are explained.

#### 3.A. SuPerHomogenization (SPH) factors

As previously introduced, due to the scaling down from assembly to pin wise calculations, the conservation of reaction rates between heterogeneous and homogeneous calculations is lost. With the intention of compensating this loss the SPH factors are employed in SPHINCS.

The SPH factor expression consists in the ratio between the homogenized flux coming from the transport code ( $\bar{\phi}^*$ ) employed for the homogenized GCs generation and the one obtained in the code with homogenized nodes ( $\bar{\phi}$ ). Eq. (1) gives the expressions for the SPH factor, where  $\zeta$  represents the SPH factor.

$$\begin{aligned} \bar{\Sigma}_{pin} \bar{\phi}_{pin} &\neq \bar{\Sigma}_{pin} \bar{\phi}_{pin}^* \\ \zeta &= \frac{\bar{\phi}_{pin}^*}{\bar{\phi}_{pin}} \\ \bar{\zeta} \bar{\Sigma}_{pin} \bar{\phi}_{pin} &= \bar{\Sigma}_{pin} \bar{\phi}_{pin}^* \end{aligned} \quad (1)$$

The SPH factors are generated at a SA level with reflective BCs prior to the core or CB calculation. Throughout an iterative procedure, pin and group wise SPH factors are obtained following the code flow depicted in Fig. 1. Once the SPH factors have converged they are introduced in the FDM calculation by multiplying the homogeneous flux.

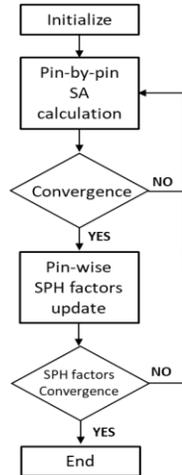


Fig. 1. SPH factors generation iterative scheme.

### 3.B. Leakage Feedback Method (LFM)

The LFM pursues the functionalization of the pin homogenized GCs with the LTRRs. This way the perturbation of the condensed GCs coming from the neighbouring assemblies can be corrected. The three group pin wise LTRRs are calculated following Eq. (2) expression. Where G denotes the fast (F), intermediate (I) or thermal energy ranges (T)

$$l_G = \frac{Q_G - \sum_{r,G} \phi_G}{\sum_{r,G} \phi_G} \quad (2)$$

This way the relative difference of the GCs can be parameterized as:

$$\frac{\Sigma_G^{core} - \Sigma_G^{SA}}{\Sigma_G^{SA}} = \alpha_G \Delta l_F + \beta_G \Delta l_R + \gamma_G \Delta l_T \quad (3)$$

And the GCs to be employed in the core calculation can thereby corrected from the original SA generated ones.

$$\Sigma_G^{core} = \Sigma_G^{SA} (1 + \alpha_G (l_F^{core} - l_F^{SA}) + \beta_G (l_I^{core} - l_I^{SA}) + \gamma_G (l_T^{core} - l_T^{SA})) \quad (4)$$

Note that in Eq. (4) two series of terms are highlighted. In red are the coefficients and in blue the parameters. Thus the LFM requires two steps to obtain the two terms. The first one, to obtain the 3G coefficients, consists of a fixed-point least-square fitting using CB generated MG GCs. Generally, if in the target core there are N different types of assemblies, the number of different CB calculations required should be at least of N(N-1).

The parameters however are generated at core level following the iterative procedure shown in Fig. 2. The parameters (which are the LTRRs) are calculated after each core iteration and the GCs updated before starting the next one.

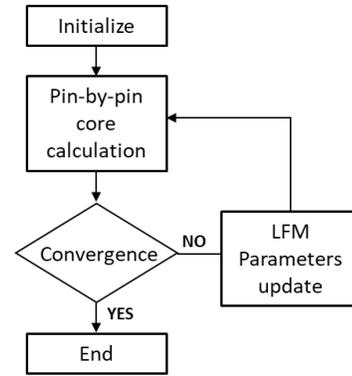


Fig. 2. LFM parameters calculation iterative scheme.

## 4. BEAVRS calculations with SPHINCS

The pin-homogenized 8-GCs are generated by 47G NTRACER calculations with the transport corrected P0 MOC option. The FAs GCs are based on the SA geometry, and for the radial reflectors are based on the fuel-reflector problems. The SPH factors corresponding to the GCs are generated by the SPHINCS calculations for the same geometry with 1x1 mesh per pin. The LFM coefficients are generated as described in Section 3. The leakage correction is not applied to the pin-wise SPH factors, nor to the reflector GCs.

The assessment of the SPHINCS solution of the benchmark can therefore be started. This assessment consists of both single assembly calculations and core calculations. The former one is a good test to check if the SPH factors are properly generated in SPHINCS. In effect, for both keff and pin power distribution the difference is zero.

The core one gives an evaluation of the convenience of using the pin wise 3G LFM. The application of the LFM results in an improvement of the results for both the eigenvalue keff and the pin power. Table II shows this upgrade. Besides the typical center-periphery tilt is smoothed as depicted in Fig. 3

Table II: Results for BEAVRS 2D core 8G SPHINCS calculation (NTRACER keff reference-1.00415)

Case	keff (keff error-pcm)	Pin Power Error (%)	
		RMS	MAX
SPH	1.00428 (12.88)	0.89	2.36
SPH + LFM	1.00421 (5.74)	0.35	1.44

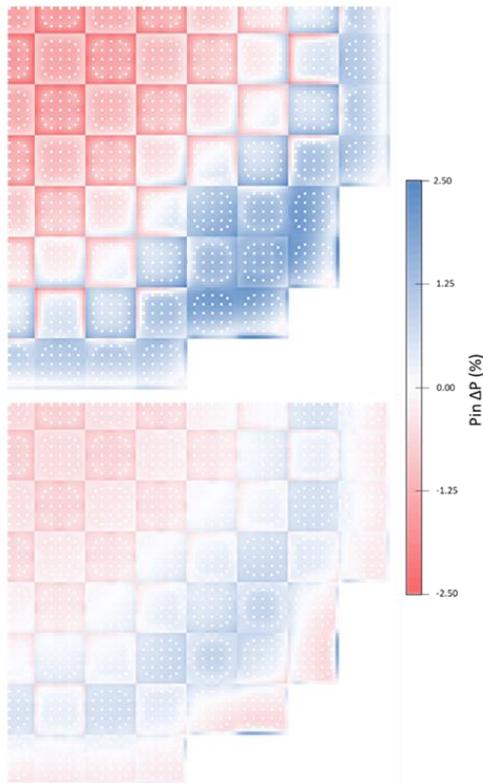


Fig. 3. Pin power error (%) distribution for 8G SPHINCS calculation with SPH factors (top) and SPH factors + LFM (bottom).

## 5. Conclusions

The validation of the SPHINCS code with the computation of the BEAVRS benchmark was satisfactorily carried out. The results obtained show a good agreement between the SP3 pin wise homogenized calculation and the heterogeneous MOC one.

In addition, the two correction methods implemented in the code have been evaluated. The SPH factor generation is accomplished as the difference between NTRACER heterogeneous results and SPHINCS show no difference whatsoever.

Leakage Feedback Method for its part has been proved to accurately correct the error introduced by computing the pin wise GCs generation using single assembly transport calculations and modify thereby the cross sections depending on the location of each assembly in the core.

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